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Set Labelling Vertices To Ensure Adjacency Coincides With Disjointness

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Abstract

Given a set of nonempty subsets of some universal set, their **intersection** graph is defined as the graph with one vertex for each set and two vertices are adjacent precisely when their representing sets have non-empty intersection. Sometimes these sets are finite, but in many well known examples like geometric graphs (including interval graphs) they are infinite. One can also study the reverse problem of expressing the vertices of a given graph as distinct sets in such a way that adjacency coincides with intersection of the corresponding sets. The sets are usually required to conform to some template, depending on the problem, to be either a finite set, or some geometric set like intervals, circles, discs, cubes etc. The problem of representing a graph as an intersection graph of sets was first introduced by Erdos [1] and they looked at minimising the underlying universal set necessary to represent any given graph. In that paper it was shown that the problem is NP complete. In this paper we study a natural variant of this problem which is to consider graphs where vertices represent **distinct** sets and adjacency coincides with **disjointness**. Although this is **nearly** the same problem on the complement graph, for specific families of graphs this is a more natural way of viewing it. The parameter we take into account is the minimum universe size possible (disregarding individual label sizes).

Keywords: Set labelling, Intersection number, Intersection graphs, Vertex labelling, Knesser graphs

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1 Introduction

Knesser graphs $KG_{n,k}$ are graphs whose vertices correspond to the k element subsets of an n element set and two vertices are adjacent precisely when their corresponding subsets are disjoint. Clearly if $n < 2k$ then the graph is an independent set of vertices. If $n = 2k$ then the graph is a matching. When $n = 2k+1$ we get the special family graph of **odd graphs** [6]. Knesser graphs are well studied [3][7][9]. Many problems on them can be solved clearly and efficiently using this set-theoretic definition. A natural question, therefore, is to try and model an arbitrary graph in this fashion. That is, come up with an underlying universal set and a choice of unique subsets to associate with each vertex such that adjacency is characterized by disjointness of the corresponding subsets. Clearly for an arbitrary graph the above choice of all identical sized subsets of a certain set will not work, because a graph defined in that manner is necessarily vertex transitive.

Our motivation to look at disjointness instead of intersection is that several well known graphs like the Petersen graph and Knesser graphs are expressed in the latter method, and the complements of these families are not well studied. Thus our choice is justified and not merely an attempt to artificially deviate from existing work.

The closely related concept is **intersection graphs** [8] for finite sets in which non-adjacency is characterized by disjointness of the corresponding subsets of underlying universal set. This was studied by Erdos et. al. [4]. In that paper, they also obtain a tight upper bound of $n^2/4$ on intersection number where the sets are not required to be distinct. In Section 6 of that paper, the authors point out that the problem in general becomes more difficult when the constraint of the distinctness is added. They, however, observe that the universal upper bound applies to that variant as well. We, thus, make inroads into an open problem posed by them obtaining some general results as well as results for some special classes of graphs. We use the slightly different framework of disjointness graphs as many well known families of graphs are defined in this way, as mentioned earlier. So for a given graph, these two labelling approaches are entirely different (except for self-complementary graphs).

For a graph with m edges and n vertices, a trivial upper bound for intersection number is m (see [2]). Alon Noga et.al. [1] derived an upper bound for any N- vertex graph as a function of maximum degree of the graph: $2e^2(d+1)^2 \ln N$ where d=maximum degree of the complement graph of G and e=base of the natural logarithm.

Since the problems of intersection and disjointness on graph representation