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Edge a-Zagreb Indices and its Coindices of Transformation Graphs

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Abstract

The edge a-Zagreb index and its coindex are defined as $Z_a(G) = \sum_{uv \in E(G)} (d^a(u) + d^a(v))$ and $\overline{Z}_a(G) = \sum_{uv \notin E(G)} (d^a(u) + d^a(v))$. In this paper, we obtain the exact expressions for the edge a-Zagreb indices and its coindices of two different transformation of graphs and their complements. Using the results obtained here, the value of F- index and its coindex for above transformation graphs are obtained.

Keywords: Zagreb index, Zagreb coindex, F-index, F-coindex

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1 Introduction

A *chemical graph* is a graph whose vertices denote atoms and edges denote bonds between those atoms of any underlying chemical structure. A *topological index* for a (chemical) graph G is a numerical quantity invariant under automorphisms of G and it does not depend on the labeling or pictorial representation of the graph. In the current chemical literature, a large number of graph-based structure descriptors (topological indices) have been put forward, that all depend only on the degrees of the vertices of the molecular graph. More details on vertex-degree-based topological indices and on their comparative study can be found in [4,5,6,9]. The topological indices are graph invariants which has been used for examing quantitative structure-property relationships (QSPR) and quantitative structure-activity relationships (QSAR) extensively in which the biological activity or other properties of molecules are correlated with their chemical structures, see [3].

For a (molecular) graph G, The *first Zagreb index* $M_1(G)$ is the equal to the sum of the squares of the degrees of the vertices, and the *second Zagreb index* $M_2(G)$ is the equal to the sum of the products of the degrees of pairs of adjacent vertices, that is, $M_1(G) = \sum\limits_{u\in V(G)} d_G^2(u) = \sum\limits_{uv\in E(G)} (d_G(u) + d_G(v))$, $M_2(G) = \sum\limits_{uv\in E(G)} d_G(u)d_G(v)$. The *first* and *second Zagreb coindices* were first introduced by Ashrafi et al. [1]. They are defined as follows: $\overline{M}_1(G) = \sum\limits_{uv\notin E(G)} (d_G(u) + d_G(v))$, $\overline{M}_2(G) = \sum\limits_{uv\notin E(G)} d_G(u)d_G(v)$.

The *forgotten topological index* was introduced by Furtula and Gutman [7], and it is defined as $F = F(G) = \sum\limits_{u\in V(G)} d_G^3(u) = \sum\limits_{uv\in E(G)} (d_G^2(u)+d_G^2(v))$. In this sequence, the *forgotten topological coindex* is defined as $\overline{F}(G) = \sum\limits_{uv\notin E(G)} (d_G^2(u)+d_G^2(v))$.

Mansour and Song [12] were introduced the *vertex a-Zagreb index,edge a-Zagreb index*, and *edge a-Zagreb Coindex*. They are defined as follows. $N_a(G) = \sum\limits_{v\in V(G)} d^a(v)$,$Z_a(G) = \sum\limits_{uv\in E(G)} (d^a(u)+d^a(v))$ and $\overline{Z}_a(G) = \sum\limits_{uv\notin E(G)} (d^a(u)+d^a(v))$. One can observe that $N_0(G) = |V(G)|$,$N_1(G) = Z_0(G) = 2|E(G)|$, $N_2(G) = Z_1(G) = M_1(G)$ and $N_3(G) = Z_2(G) = F(G)$. Similarly, $\overline{Z_0}(G) = 2\binom{|V(G)|}{2} - 2|E(G)|$, $\overline{Z}_1(G) = \overline{M}_1(G)$, and $\overline{Z}_2(G) = \overline{F}(G)$.

Khalifeh et al. [11] obtained the first and second Zagreb indices of the Cartesian, join, composition, disjunction and symmetric difference of two graphs. Ashrafi et al. [1] obtained the first and second Zagreb coindices