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Distance Antimagic Labeling of the Ladder Graph

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Abstract

Let G be a graph of order n. Let $f : V(G) \longrightarrow \{1, 2, \ldots, n\}$ be a bijection. The weight $w_f(v)$ of a vertex with respect to f is defined by $w_f(v) = \sum_{x \in N(v)} f(x)$. The labeling f is said to be distance antimagic if $w_f(u) \neq w_f(v)$ for every pair of vertices $u, v \in V(G)$. If the graph G admits such a labeling then G is said to be a distance antimagic graph. In this paper we investigate the existence of distance antimagic labeling in the ladder graph $L_n \cong P_2 \square P_n$.

Keywords: Distance antimagic graphs, antimagic labeling, arbitrarily distance antimagic.
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1 Introduction

By a graph $G = (V, E)$, we mean a finite undirected graph without loops, multiple edges or isolated vertices. For graph theoretic terminology we refer

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to West [7].

Most of the graph labeling methods trace their origin to the concept of β-valuation introduced by Rosa [6]. For a general overview of graph labeling we refer to the dynamic survey by Gallian [2].

Let G be a graph of order n. Let $f : V(G) \longrightarrow \{1, 2, \ldots, n\}$ be a bijection. The weight $w_f(v)$ of a vertex v is defined by $w_f(v) = \sum_{x \in N(v)} f(x)$ where $N(v)$ is the open neighbourhood of the vertex v. The labeling f is said to be distance antimagic if $w_f(u) \neq w_f(v)$ for every pair of vertices $u, v \in V(G)$. If the graph G admits such a labeling, then G is said to be a distance antimagic graph. Many classes of graphs are known to be distance antimagic. For details one may refer to Gallian [2] and Arumugam *et al.* [1]. In [5] Kamatchi and Arumugam posed the following problem:

Problem 1.1 *If G is a distance antimagic graph, is it true that $G + K_1$ and $G + K_2$ are distance antimagic?*

In [3] Handa *et al.* solved the problem in the affirmative. They introduced the concept of arbitrarily distance antimagic labeling as a tool to study distance antimagic labeling of join of two graphs.

Definition 1.2 A graph G of order n is said to be arbitrarily distance antimagic if there exists a bijection $f : V \longrightarrow \{1, 2, \ldots, n\}$ such that $w_{f_k}(u) \neq w_{f_k}(v)$ for any two distinct vertices u and v and for any $k \geq 0$. The labeling f with this property is called an arbitrarily distance antimagic labeling of G.

The following results are proved in [3].

Proposition 1.3 *[3] Any r-regular distance antimagic graph G is arbitrarily distance antimagic.*

Theorem 1.4 *[3] Let f be a distance antimagic labeling of a graph G of order n. If $w_f(u) < w_f(v)$ whenever $deg(u) < deg(v)$, then G is arbitrarily distance antimagic.*

Proposition 1.5 *[3] Let G_1 and G_2 be two graphs of order n_1 and n_2 with arbitrarily distance antimagic labelings f_1 and f_2 respectively, and let $n_1 \leq n_2$. Let $x \in V(G_1)$ be the vertex with lowest weight under f_1 and $y \in V(G_2)$ be the vertex with highest weight under f_2. If*

$$w_{f_1}(x) + \sum_{i=1}^{n_2}(n_1 + i) > w_{f_2}(y) + \Delta(G_2)n_1 + \sum_{i=1}^{n_1} i \tag{1}$$

then $G_1 + G_2$ is distance antimagic.