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On self-centeredness of tensor product of some graphs

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Abstract

A simple connected graph G is said to be a self-centered graph if every vertex has the same eccentricity. Tensor product of self-centered graphs may not be a self-centered graph. In this paper we study self-centeredness property of tensor product of cycles with themselves and other graphs, complete graphs with themselvesand other self-centered graphs, wheel graphs with themselves and other self-centered graphs, and square of cycles with themselves.

Keywords: Eccentricity, Radius, Diameter, Self-centered graph, Tensor Product.

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1 Introduction

The concept of self-centered graphs and its applications have been widely discussed in [2]. One can find the application of self-centered graphs in facility location problems where the resources (facility) must be located at central node for its efficient use. Tensor product of graphs was introduced by Whitehead and Russell [9]. Modeling of internet graphs is one of the applications of tensor product of graphs [4]. In this paper we consider simple and connected graphs only. For any two vertices u and v in a graph G, the length of a shortest $u - v$ path is known as the *distance* between u and v and is denoted by $d(u, v)$. The *eccentricity* of a vertex v in G, denoted by $e(v)$, is defined as the distance between v and a vertex farthest from v in G, i.e., $e(v) = \max\{d(v, u) : u \in V(G)\}$. The radius $rad(G)$ and diameter $diam(G)$ of graph G are respectively the minimum and maximum eccentricity of the vertices, i.e., $rad(G) = \min\{e(v) : v \in V(G)\}$ and $diam(G) = \max\{e(v) : v \in V(G)\}$. A graph G is called a *self-centered* graph if eccentricity of every vertex is the same. Further, G is called a *d-self-centered* graph if the eccentricity of every vertex is d. The k^{th} power G^k of a graph G is the graph on the same set of vertices as G and any two distinct vertices are adjacent if and only if the distance between them is at most k in G.

Tensor product of n graphs $G_1, \ldots, G_n$, denoted by $G_1 \otimes G_2 \otimes \ldots \otimes G_n$, is the graph G with vertex set $V(G) = \{(x_1, x_2, \ldots, x_n) : x_i \in V(G_i)\}$ and two vertices $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ are adjacent in G if and only if $x_i \sim y_i$ in G_i for every $1 \le i \le n$. The distance between any two vertices x and y in G is given by [4],

$$d(x, y) = \min\{k | \text{each } G_i \text{ has an } x_i - y_i \text{ walk of length } k \text{ for } 1 \le i \le n\}. \quad (1)$$

Any two walks in a graph are said to have the *same parity* if the difference of their lengths is even; otherwise they have *opposite parity*. For any two vertices x and y (not necessarily distinct) in a graph G, the *upper distance* [1] between x and y, denoted by $D(x, y)$, is the minimum length of an $x - y$ walk whose parity differs from a shortest $x - y$ path. If no such walk exists, then we take $D(x, y) = \infty$. The *upper eccentricity* [1] $E(x)$ of a vertex $x \in V(G)$ is defined as $E(x) = \max\{D(x, y) : y \in V(G), \text{ and y need not be different from } x\}$.

For any two vertices x and y in a bipartite graph G there is no $x - y$ walk in G which is opposite parity with a shortest $x - y$ path in G. So we have the following result.