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Adjacency Matrix of a Semigraph

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Abstract

Semigraph was defined by Sampathkumar as a generalization of a graph. In this paper the adjacency matrix which represents semigraph uniquely and a characterization of such a matrix is obtained. An algorithm to construct the semigraph from a given square matrix, if semigraphical is given. Some properties of adjacency matrix of semigraph are studied. A sufficient condition for eigen values to be real is also obtained.

Keywords: Semigraph, adjacency matrix of semigraph, semigraphical matrix, eigen values of semigraph.

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1 Introduction

Semigraph was introduced by Sampathakumar[4] as a generalization of graph.

Definition 1.1 *Semigraph* is an ordered pair of two sets V and X, where V is a non-empty set whose elements are called vertices of G and X is a set of n-tuples of distinct vertices, called edges of G, for various n (n at least 2) satisfying the following conditions:

(i) (SG1) - Any two edges have at most one vertex in common.

(ii) (SG2) - Two edges $(u_1, u_2, \cdots, u_n)$ and $(v_1, v_2, \cdots, v_m)$ are considered to be equal if
  (a) m = n and
  (b) either $u_i = v_i$ for $i = 1, 2, \cdots, n$ or $u_i = v_{n+1-i}$, for $i = 1, 2, \cdots, n$.

Thus the edge $(u_1, u_2, \cdots, u_m)$ is same as $(u_m, u_{m-1}, \cdots, u_1)$.

Example 1.2 Let $G = (V, X)$ be a semigraph where $V = \{v_1, v_2, \cdots, v_9\}$ and $X = \{(v_1, v_2, v_3, v_4), (v_4, v_5), (v_1, v_6, v_5), (v_4, v_6, v_7), (v_5, v_7), (v_2, v_6), (v_5, v_9)\}$.

For the edge $e = (u_1, u_2, \cdots, u_n)$, u_1 and u_n are called the *end* vertices of e and $u_2, u_3, \cdots, u_{n-1}$ are called the *middle* vertices of e. If a vertex is an end vertex of an edge and a middle vertex of another edge then such a vertex is called a middle-end vertex of G and a vertex that belongs to no edge is called an isolated vertex. For the semigraph G in Example 1.2, v_1, v_4, v_5, v_7 and v_9 are the end vertices; v_3 is a middle vertex; v_2 and v_6 are the middle-end vertices and v_8 is an isolated vertex.

Two vertices in a semigraph are said to be ***adjacent*** if they belong to the same edge and are said to be ***consecutively adjacent*** if in addition they are consecutive in order as well. For example, in the semigraph G of Example 1.2, the vertices v_1 and v_4 are adjacent while the vertices v_2 and v_3 are consecutively adjacent. ***Cardinality*** of an edge e in a semigraph G is the number of vertices lying on that edge. In the semigraph G of Example 1.2, cardinality of $e = (v_1, v_2, v_3, v_4)$, denoted as $|e|$, is 4.

In this paper, we first define adjacency matrix of a semigraph and prove its uniqueness. An algorithm to construct the semigraph from adjacency matrix

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