# Minimum Geodetic Fuzzy Subgraph

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## Abstract

The concept of Minimum geodetic subgraphs in Graph theory was introduced by G.Chartrand,F.Harary and P.Zhang in 2001.In this paper,the idea is extended to fuzzy graphs using geodesic distance.An upper and lower bound for geodesic number of fuzzy graphs is obtained using which it is established that the geodesic number of a fuzzy graph exceeds or is equal to that of its minimum geodetic fuzzy subgraph. A necessary condition for the geodesic number of a fuzzy graph to be 2 depending on the nodes of its minimum geodetic fuzzy subgraph is obtained and a study on the minimum geodetic fuzzy subgraph of a fuzzy tree is conducted.

*Keywords:*  geodesic distance, geodesic cover, geodesic basis, geodesic number.

# 1    Introduction

Zadeh in 1965 [23] brought the concept of fuzzy sets into existence which gave a platform for describing the uncertainties prevailing in day-today life situations. Later on, the theory of fuzzy graphs was developed by Rosenfeld in the year 1975 [16]along with Yeh and Bang [22]. Rosenfeld also obtained the fuzzy analogue of several graph theoretic concepts like paths,cycles,trees and connectedness along with some of their properties [16] and the concept of fuzzy trees [14],automorphism of fuzzy graphs [2], fuzzy interval graphs [11],cycles and cocycles of fuzzy graphs [12] etc has been established by several authors during the course of time. Fuzzy groups and the notion of a metric in fuzzy graphs was introduced by Bhattacharya [1]. The concept of strong arcs [5] was introduced by Bhutani and Rosenfeld in the year 2003. The definition of fuzzy end nodes and some of its properties were established by the same authors in [3]. Several other significant works on fuzzy graphs can be found in [10,15,20]. The concept of geodesic distance was introduced by Bhutani and Rosenfeld in [4]. Balazs Feil and Janos Abonyi in 2007 applied geodesic distance in fuzzy clustering [7]. Further studies based on the geodesic distance were carried out by Sameena and Sunitha in [17,18,19]. Later on, using this geodesic distance, Suvarna and Sunitha in [21] introduced the concept of geodesic iteration number and geodesic number of a fuzzy graph and studied some of the properties. The same concepts using $\mu$-distance was introduced by Linda and Sunitha in [9]. Linda and Sunitha in [9] also introduced the concepts of $g$-periphery, $g$-eccentric fuzzy graph, $g$-boundary node and $g$-interior node in a fuzzy graph and studied some of their properties. Fuzzy detour $g$-distance was introduced by Linda and Sunitha in [9] in which the authors also introduced fuzzy detour $g$-boundary nodes and fuzzy detour $g$-interior nodes in fuzzy graphs.
In Graph theory, the concept of minimum geodetic subgraphs was introduced by Gary Chartrand, Frank Harary and Ping Zhang in 2001 and some of the properties satisfied by them can be seen in [6].
In this paper, using geodesic distance, the concept of minimum geodetic fuzzy subgraph is introduced and established some of the properties satisfied by them. The minimum geodetic fuzzy subgraph of a fuzzy tree is the fuzzy subgraph induced by its fuzzy end nodes.

# 2    Preliminaries

A fuzzy graph [13] is a triplet $G : (V, \sigma, \mu)$ where $V$ is vertex set, $\sigma$ a fuzzy subset of $V$ and $\mu$ a fuzzy relation on $\sigma$ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$,