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Unitary Cayley Meet Signed Graphs

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Abstract

A *signed graph* S is a graph in which every edge receive either '+' or '-' called the *signs* of the edges. The *unitary Cayley graph* X_n is a graph with vertex set Z_n, the integers modulo n, where n is a positive integer greater than 1. Two vertices x_1 and x_2 are adjacent in the unitary Cayley graph if and only if their difference is in U_n, where U_n denotes set of all units of the ring Z_n. The properties of balancing and clusterability of unitary Cayley meet signed graph $S_n^\wedge$ are discussed. Apart from this, the canonically consistency of $S_n^\wedge$ is determined when n has at most two distinct odd prime factors. Sign-compatibility has been worked out for these graphs as well.

Keywords: Signed graph, balance, clustering, $\mathcal{C}$-consistency, unitary Cayley signed graph

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1 Introduction

We use standard terminology of graph theory from the books of Harary [11] and West [20], where a graph is considered to be simple, finite and loopless. For the signed graphs Zaslavsky [21], [22] is followed.

1.1 Some basic notions and notations

A *signed graph* S (*see* [10]) is an ordered pair $S = (S^u, \sigma)$, where $S^u = (V, E)$ is a graph called the *underlying graph* of S and $\sigma : E \to \{+, -\}$ is a function from the edge set E of S^u into the set $\{+, -\}$, called the *signature* (or *sign* in short) of S. The *negation* of a signed graph S is a signed graph which is obtained from S by interchanging the sign of each edge from '+' to '-' and vice-versa.

A *positive* (*negative*) *section* of a subsigned graph S' of a signed graph S is defined as a maximal edge-induced connected subsigned graph in S which consisting only the positive (negative) edges of S.

An *isomorphism of graphs* G_1 and G_2 is a bijection between the vertex sets of G_1 and G_2 $f : V(G_1) \longrightarrow V(G_2)$ such that any two vertices x and y of G_1 are adjacent in G_1 if and only if $f(x)$ and $f(y)$ are adjacent in G_2. This bijection is called "*edge-preserving bijection*". Two signed graphs S_1 and S_2 are said to be isomorphic if there is an isomorphism between their underlying graphs such that it preserves edge signs.

A positive cycle in a signed graph S is a cycle which contains an even number of negative edges. A signed graph S is a *balanced signed graph* if every cycle in S is positive (*see* [10]).

1.2 Balance in a signed graph

The *partition criterion* to characterize the balancing property of a signed graph was given by Harary [10].

Theorem 1.1 [10] *A signed graph S is balanced if and only if its vertex set $V(S)$ can be partitioned into two subsets V_1 and V_2, one of them possibly empty, such that every positive edge joins two vertices in the same subset and*