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Transitivity Model on Signed Graphs

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Abstract

In this paper, we generalize the already established iterated local transitivity model for online social networks in signed networks. In this model, at each time step t and for already existing vertex x, a new vertex(clone) x' is added which joins to the neighbors of x. The sign of new edge xx' is the marking on x. We also discuss the properties such as balance, clusterability, sign-compatibility and consistency. The signed networks focus on the type of relations (friendship and enmity) between the vertices(members of online social network). The ILT model for signed network gives an insight on how the network reacts to the addition of clone vertex. Also the properties like balance and clusterability helps establish a natural balance in society by providing a possible formation of group of vertices in society for a peaceful co-existence and smooth functioning of social system.

Keywords: social network, signed social network, local transtivity model, marked singed graph, neighborhood, balance, sign-compatibility, clusterability, algorithm.

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1 Introduction

In this paper, we extend the work on the theoretic aspect of friendship and enmity for online social networks which grow very fast. We give sign to edges where a positive sign indicates a friendly relation and a negative sign a hostile relation. Further, a positive vertex indicates that a person befriends most of his/her neighbors and a negative vertex states a has hostile relations with their acquaintances.

A *signed graph* [4] is an ordered pair $\Sigma = (\Sigma^u, \sigma)$, where Σ^u is a graph $G = (V, E)$, called the underlying graph of Σ and $\sigma : E \to \{+, -\}$ is a function from the edge set E of Σ^u into the set $\{+, -\}$, called the *signature* (or *sign* in short) of Σ. A signed graph is *all-positive* (respectively, *all-negative*) if all its edges are positive (negative); further, it is said to be *homogeneous* if it is either all-positive or all-negative and *heterogeneous* otherwise. The *positive* (*negative*) *degree* of a vertex $v \in \Sigma$ denoted by $deg^+(v)(deg^-(v))$ is the number of positive (negative) edges incident on the vertex v and $deg(v) = deg^+(v) + deg^-(v)$.

Let v be an arbitrary vertex of a graph G. We denote the set consisting of all the vertices of G adjacent to v by $N(v)$. This set is called the *open neighborhood* set of v and sometimes we call it as *open neighborhood* of v. The set consisting of all the vertices of G adjacent to v along with v itself is called the *closed neighborhood* set or *closed neighborhood* of v. It is denoted by $N[v]$.

A *marked signed graph* is an ordered pair $\Sigma_\mu = (\Sigma,\ \mu)$ where $\Sigma = (\Sigma^u, \sigma)$ is a signed graph and $\mu : V(\Sigma^u) \to \{+, -\}$ is a function from the vertex set $V(\Sigma^u)$ of Σ^u into the set $\{+, -\}$, called a *marking* of Σ. Henceforth the vertex receiving '+' mark will be called *positive vertex* and the vertex receiving '-' mark is called negative vertex. In this model the marking on a vertex denotes its nature of bond with its neighbors. A positive vertex indicates that a person befriends most of its neighbors and a negative vertex states that it has hostile relations with its acquaintances. A new clone vertex added makes his relations according to these markings of already existing vertices.

A *cycle* in a signed graph S is said to be *positive* if the product of the signs of its edges is positive or, equivalently, if the number of negative edges in it is even. A cycle which is not positive is said to be *negative* A signed graph is *balanced* if all its cycles are positive. The partition criterion to characterize the balance property of a signed graph is given by Harary [2]. The balance in ILT model for signed networks partitions our network into two groups so

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