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Topological methods for the existence of a rainbow matching

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Abstract

We present recent results regarding rainbow matchings in bipartite graphs. Using topological methods we address a known conjecture of Stein and show that if $K_{n,n}$ is partitioned into n sets of size n, then a partial rainbow matching of size $2n/3$ exists. We generalize a result of Cameron and Wanless and show that for any n matchings of size n in a bipartite graph with $2n$ vertices there exists a full matching intersecting each matching at most twice. We show that any n matchings of

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size approximately $3n/2$ have a rainbow matching of size n. Finally, we show the uniqueness of the extreme case for a theorem of Drisko and provide a generalization of Drisko's theorem.

Keywords: partial rainbow matching, full rainbow matching, bipartite graph, Ryser-Brualdi Conjecture, Stein's conjecture

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1 The case of n sets of size n

Given sets $F_1, F_2, \ldots, F_n$ of edges in a graph, a *(partial) rainbow matching* is a partial choice function on the F_is whose range is a matching. If the rainbow matching represents all of the F_is, then it is a *full rainbow matching.*

A known conjecture of Ryser and Brualdi [10,18,19] states that any n matchings $F_1, F_2, \ldots F_n$ of size n that form a partition of $K_{n,n}$ have a partial rainbow matching of size $n-1$. The best result so far towards proving this conjecture belongs to Hatami and Shor [14] who showed that in any such case a partial rainbow matching of size $n - 11\log_2^2 n$ exists.

The Ryser-Brualdi conjecture can be generalized in different ways. We may ease the requirement that the matchings $F_1, F_2, \ldots F_n$ form a partition of $K_{n,n}$ [1]:

Conjecture 1.1 *Any n matchings of size n in a bipartite multigraph have a partial rainbow matching of size $n-1$.*

The best result in this direction in due to Woolbright [20] and Brouwer, de Vries and Wieringa [9] who showed (essentially) that a rainbow matching of size $n - \lfloor\sqrt{n}\rfloor$ exists.

Given sets $F_1, F_2, \ldots F_n$, if a matching of size n in their multiset union intersects each F_i at most twice, we call it a *half-rainbow matching.* Cameron and Wanless [11] showed that in the Ryser-Brualdi setup (that is, when the matchings $F_1, F_2, \ldots F_n$ form a partition of $K_{n,n}$) a half-rainbow matching of size n exists. We generalize the Cameron-Wanless result to the case of any n matchings, namely,

Theorem 1.2 *[3] Any n matchings of size n in a bipartite multigraph with $2n$ vertices have a half-rainbow matching.*

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