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Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 63 (2017) 485–495 www.elsevier.com/locate/endm

Energy and Wiener index of Total Graph over Ring $\mathbb{Z}_\mathbf{n}$

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Abstract

Let $\mathbb{Z}_n$ be a commutative ring with $Z(\mathbb{Z}_n)$ its set of zero-divisors. In this paper, we study the total graph of $\mathbb{Z}_n$, denoted by $T(\Gamma(\mathbb{Z}_n))$. It is the (undirected) graph with all elements of $\mathbb{Z}_n$ as vertices, and for distinct $x, y \in \mathbb{Z}_n$, the vertices x and y are adjacent if and only if $x + y \in Z(\mathbb{Z}_n)$. We study the Energy, Laplacian matrix, Laplacian energy, Distance energy and Wiener index of the total graph of $\mathbb{Z}_n$, where n is the product of primes. We also find the relation among all energies. Moreover, we have given MATLAB coding of our calculations for Energy and Wiener index.

Keywords: Commutative ring, Total graph, Adjacency matrix, Laplacian matrix, Laplacian energy, Distance energy, Wiener index.

[1] I thanks to my guide who always help me to correct my mistakes.

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https://doi.org/10.1016/j.endm.2017.11.047

1 Introduction

Let $\mathbb{Z}_n$ be the commutative ring of residue classes with respect to modulo n and $Z(\mathbb{Z}_n)$, the set of zero-divisors of $\mathbb{Z}_n$, i. e., $Z(\mathbb{Z}_n)= \{x \in \mathbb{Z}_n : xy = 0,$ for some $y \in \mathbb{Z}_n\}$. Then zero-divisor graph of $\mathbb{Z}_n$ is an undirected graph $\Gamma(\mathbb{Z}_n)$ with vertex set $Z(\mathbb{Z}_n)$ such that distinct vertices x and y of $Z(\mathbb{Z}_n)$ are adjacent if and only if $xy = 0$. D. F. Anderson and Badawi [2] introduced the total graph of R, denoted by $T(\Gamma(R))$, as the graph with all elements of R as vertices, and for distinct $x, y \in R$, the vertices x and y are adjacent if and only if $x + y \in Z(R)$. They studied some graphical parameters of this graph such as diameter and girth. In addition, they studied some special subgraphs of $T(\Gamma(R))$, and study the total graph based on these subgraphs. In [1] Akbari and et al., proved that the total graph is a Hamiltonian graph if it is connected. In [7], Maimani and et al. have given the necessary and sufficient condition for the planarity of total graphs of a commutative ring. They have also characterized that all rings with total graphs have genus 1.

Here, for a graph G, the set of vertices is denoted by $V(G)$ and the set of edges by $E(G)$. In the graph G if $x \in V(G)$, then the degree of x, denoted by $deg(x)$, is the number of edges of G incident with x. A graph is almost connected if there exists a path between any two non-isolated vertices. A complete graph is a graph in which each pair of distinct vertices is joined by an edge. We denote the complete graph with n vertices by K_n. For a positive integer r, a graph is said to be r-partite if vertex-set is partitioned into r disjoint subsets in such a way that the two end vertices for each edge lie in distinct partitions. A complete r-partite graph is one in which each vertex is joined to every vertex that is not in the same partition. The complete 2-partite graph (also called the complete bipartite graph) with exactly two partitions of size m and n, is denoted by $K_{m,n}$.

2 Energies of $T(\Gamma(\mathbb{Z}_n))$

The adjacency matrix of $T(\Gamma(\mathbb{Z}_n))$ is defined by a matrix $A = [a_{ij}]$, where $a_{ij} = 1$, if v_i and v_j joined by an edge for any vertices v_i and v_j of $T(\Gamma(\mathbb{Z}_n))$ and $a_{ij} = 0$, otherwise. In graph theory, the energy of a graph is the sum of absolute values of the eigenvalues of the adjacency matrix. We know that here A is a real symmetric matrix so all the eigenvalues are real and $tr(A) = 0$.

Theorem 2.1 *If $p = 2$ and q is an odd prime, then energy of $T(\Gamma(\mathbb{Z}_{2q}))$ is $4(q - 1)$.*