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On the Grundy number of Cameron graphs

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Abstract

The Grundy number of a graph is the maximal number of colors attained by a first-fit coloring of the graph. The class of Cameron graphs is the Seidel switching class of cographs. In this paper we show that the Grundy number is computable in polynomial time for Cameron graphs.

Keywords: Grundy number, Cameron graphs, coloring.

1 Introduction

A proper coloring of a graph is a partition of its vertices into independent sets. We refer to the sets in the partition as color classes, or simply as colors. The chromatic number of a graph G, denoted as $\chi(G)$, is the minimal number of colors used in a proper coloring.

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Definition 1.1 Let $\{C_1, \ldots, C_k\}$ be the color classes of a proper coloring of G. The coloring is a first-fit coloring if each vertex in color class C_j has at least one neighbor in every color class C_i with $i < j$.

The maximal number of color classes in a first-fit coloring is called the Grundy number of G. We denote the Grundy number as $\Gamma(G)$. Notice that, if $C_1, \ldots, C_k$ are the color classes of a first-fit coloring, then for each i, C_i is a maximal independent set in the subgraph induced by

$$\bigcup_{j=i}^{k} C_j.$$

This property characterizes first-fit colorings.

A graph is a cograph if it has no induced P_4, that is a path with four vertices. Cographs are the graphs that are closed under unions and joins. It is easily seen that in every cograph, every maximal independent set meets every maximal clique. This property characterizes the class of cographs. By means of this characterization, Christen and Selkow prove the following theorem. We give a different proof.

Theorem 1.2 *When G is a cograph,*

$$\Gamma(G) = \omega(G) = \chi(G).$$

Proof. Let G be a cograph. If G is the union of two smaller cographs, G_1 and G_2, then

$$\Gamma(G) = \max \{ \ \Gamma(G_1), \ \Gamma(G_2) \ \}. \tag{1}$$

Assume that G is the join of two smaller cographs, G_1 and G_2. Then any independent set has vertices only in one of the two graph G_1 or G_2. It follows that

$$\Gamma(G) = \Gamma(G_1) + \Gamma(G_2). \tag{2}$$

Notice that the clique number, and also the chromatic number, of G, satisfy recurrences similar to (1) and (2). Since the above exhausts all alternatives, this proves the theorem. $\square$

Definition 1.3 Let G be a graph and let $S \subseteq V(G)$ be a subset of vertices of G. The Seidel switch with respect to S is the graph obtained from G by