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Signed Zero-Divisor Graph

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Abstract

Let R be a finite commutative ring with unity $(1 \neq 0)$ and let $Z(R)^*$ be the set of non-zero zero-divisors of R. We associate a (simple) graph $\Gamma(R)$ to R with vertices as elements of R and for distinct x, $y \in R$, the vertices x and y are adjacent if and only if $xy = 0$. Further, its signed zero-divisor graph is an ordered pair $\Gamma_\Sigma(R) := (\Gamma(R), \sigma)$, where for an edge ab, $\sigma(ab)$ is '+' if $a \in Z(R)^*$ or $b \in Z(R)^*$ and '−' otherwise. This paper aims at gaining a deeper insight into signed zero-divisor graph by investigating properties like, balancing, clusterability, sign-compatibility and consistency.

Keywords: finite commutative ring, zero-divisors, signed graph, negation signed graph, balancing, clusterability, sign-compatible, consistent.

1 Introduction

The idea of a zero-divisor graph of a commutative ring R was introduced by I. Beck [1], where he was mainly interested in colorings of R. They let all

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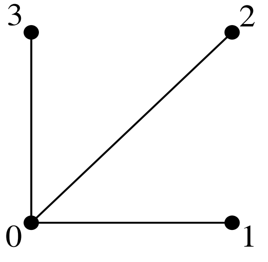
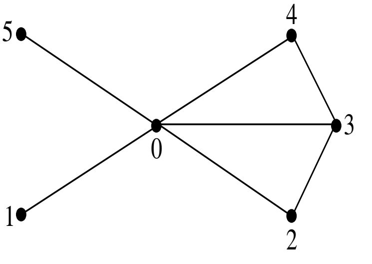
elements of R be vertices and had distinct vertices x and y adjacent if and only if $xy = 0$. The zero-divisor graphs $\Gamma(\mathbb{Z}_4)$ and $\Gamma(\mathbb{Z}_6)$ are shown in Figure 1.



(a) $\Gamma(\mathbb{Z}_4)$



(b) $\Gamma(\mathbb{Z}_6)$

Fig. 1. Zero-Divisor Graphs

Given a graph G, a *signature* σ on G is a mapping that assigns to each edge of G either a positive or a negative sign. The graph G equipped with a signature σ is called a *signed graph*, denoted by $S := (G, \sigma)$, where $G = (V, E)$ is an underlying graph and $\sigma : E \longrightarrow \{+, -\}$ is the signature that labels each edge of G either by '+' or '−'. The edge which receives the positive (respectively, negative) sign is called a *positive* (respectively, *negative*) edge. A signed graph is an *all-positive* (respectively, *all-negative*) if all its edges are positive (respectively, negative); further, it is said to be *homogeneous* if it is either an all-positive or an all-negative and *hetrogeneous* otherwise. The negation $\eta(S)$ of a signed graph S is a signed graph obtained from S by negating the sign of every edge of S.

One of the fundamental concepts in the theory of signed graph is that of *balance*. Harary [8] introduced the concept of *balanced signed graphs* for the analysis of social networks, in which a positive edge stands for a positive relation and a negative edge represents a negative relation. They have been rediscovered many times because they come up naturally in many unrelated areas.

The following is the characterization of a balanced signed graph from Cartwright and Harary [3].

Lemma 1.1 *[3](Structure Theorem) An signed graph S is balanced if and only if its vertex set can be partitioned into two subsets V_1 and V_2, one of them may be empty, such that any edge joining two vertices within the same subset is positive, while any edge joining two vertices in different subsets is negative.*

The social groups corresponding to V_1 and V_2 in Lemma 1.1 may be re-