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Minimum density of identifying codes of king grids [1]

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Abstract

A set $C \subseteq V(G)$ is an *identifying code* in a graph G if for all $v \in V(G)$, $C[v] \neq \emptyset$, and for all distinct $u, v \in V(G)$, $C[u] \neq C[v]$, where $C[v] = N[v] \cap C$ and $N[v]$ denotes the closed neighbourhood of v in G. The minimum density of an identifying code in G is denoted by $d^*(G)$. In this paper, we study the density of king grids which are strong product of two paths. We show that for every king grid G, $d^*(G) \geq 2/9$. In addition, we show this bound is attained only for king grids which are strong products of two infinite paths. Given $k \geq 3$, we denote by $\mathcal{K}_k$ the (infinite) king strip with k rows. We prove that $d^*(\mathcal{K}_3) = 1/3$, $d^*(\mathcal{K}_4) = 5/16$, $d^*(\mathcal{K}_5) = 4/15$ and $d^*(\mathcal{K}_6) = 5/18$. We also prove that $\frac{2}{9} + \frac{8}{81k} \leq d^*(\mathcal{K}_k) \leq \frac{2}{9} + \frac{4}{9k}$ for every $k \geq 7$.

Keywords: Identifying code, King grid, Discharging Method.

1 Introduction

Let G be a graph. The *neighbourhood* of a vertex v of G, denoted by $N(v)$, is the set of vertices adjacent to v in G, and the *closed neighbourhood* of v is the

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set $N[v] = N(v) \cup \{v\}$. Given a set $C \subseteq V(G)$, let $C[v] = N[v] \cap C$. We say that C is an *identifying code* of G if $C[v] \neq \emptyset$ for all $v \in V(G)$, and $C[u] \neq C[v]$ for all distinct $u, v \in V(G)$. Clearly, a graph G has an identifying code if and only if it contains no *twins* (vertices $u, v \in V(G)$ with $N[u] = N[v]$).

Let G be a (finite or infinite) graph with bounded maximum degree. For any non-negative integer r and vertex v, we denote by $B_r(v)$ the ball of radius r in G centered at v, that is $B_r(v) = \{x \mid \text{dist}(v, x) \leq r\}$. For any set of vertices $C \subseteq V(G)$, the *density* of C in G, denoted by $d(C, G)$, is defined by

$$d(C,G) = \limsup_{r \to +\infty} \frac{|C \cap B_r(v_0)|}{|B_r(v_0)|} ,$$

where v_0 is an arbitrary vertex in G. The infimum of the density of an identifying code in G is denoted by $d^*(G)$. Observe that if G is finite, then $d^*(G) = |C^*|/|V(G)|$, where C^* is a minimum-size identifying code of G.

The problem of finding low-density identifying codes was introduced in [13] in relation to fault diagnosis in arrays of processors. Particular interest was dedicated to grids as many processor networks have a grid topology. Many results have been obtained on square grids [4,1,10,2,12], triangular grids [13,11], and hexagonal grids [6,8,9]. In this paper, we study *king grids*, which are strong products of two paths. The *strong product* of two graphs G and H, denoted by $G \boxtimes H$, is the graph with vertex set $V(G) \times V(H)$ and edge set :

$$\begin{aligned} E(G \boxtimes H) = \quad & \{(a,b)(a,b') \mid a \in V(G) \text{ and } bb' \in E(H)\} \\ & \cup \{(a,b)(a',b) \mid aa' \in E(G) \text{ and } b \in V(H)\} \\ & \cup \{(a,b)(a',b') \mid aa' \in E(G) \text{ and } bb' \in E(H)\}. \end{aligned}$$

The *two-way infinite path*, denoted by $P_{\mathbb{Z}}$, is the graph with vertex set $\mathbb{Z}$ and edge set $\{\{i, i+1\} \mid \in \mathbb{Z}\}$, and the *one-way infinite path*, denoted by $P_{\mathbb{N}}$, is the graph with vertex set $\mathbb{N}$ and edge set $\{\{i, i+1\} \mid i \in \mathbb{N}\}$. A *path* is a connected subgraph of $P_{\mathbb{Z}}$. For every positive integer k, P_k is the subgraph of $P_{\mathbb{Z}}$ induced by $\{1, 2, \ldots, k\}$. A *king grid* is the strong product of two (finite or infinite) paths. The *plane king grid* is $\mathcal{G}_K = P_{\mathbb{Z}} \boxtimes P_{\mathbb{Z}}$, the *half-plane king grid* is $\mathcal{H}_K = P_{\mathbb{Z}} \boxtimes P_{\mathbb{N}}$, the *quarter-plane king grid* is $\mathcal{Q}_K = P_{\mathbb{N}} \boxtimes P_{\mathbb{N}}$, and the *king strip of height* k is $\mathcal{K}_k = P_{\mathbb{Z}} \boxtimes P_k$.

In 2001, Cohen et al. [7] proved that $d^*(\mathcal{G}_K) \geq 2/9$. In 2002, Charon et al. [3] obtained an optimal identifying code with density 2/9. They provided the tile depicted in Fig. 1, which generates a periodic tiling of the plane with periods $(0, 6)$ and $(6, 0)$, yielding an identifying code C_∞ of the bidimensional infinite king grid with density $\frac{2}{9}$.

In this paper, using the Discharging Method (see Section 3 of [11] for a