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A column generation approach for the strong network orientation problem [1]

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Abstract

In this study, an aggregated flow formulation and a column generation strategy are proposed for the Strong Network Orientation Problem (SNOP) that consists in setting an orientation for each edge in a given graph, such that the resulting digraph is strongly connected and the total travel distance between all pairs of vertices is minimized. SNOP is NP-hard and finds application in urban networks.

Keywords: Network design, road networks, strong connectivity, column generation.

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1 Introduction

Let $G = (V, E)$ be a simple connected undirected graph, with $|V| = n$ vertices and $|E| = m$ edges. Each vertex corresponds to a street intersection or a street end while an edge corresponds to a street segment between two vertices. The cost $c_{ij} > 0$ of an edge $\{i, j\} \in E$ may define its travel distance. The Strong Network Orientation Problem (SNOP) consists in setting an orientation for each edge such that the resulting digraph is strongly connected (there is a path between all pairs of vertices) and the total travel distance between all pairs of vertices is minimized. This problem is NP-hard [1] and finds tactical applications in urban networks.

The pioneering study [3] states the conditions for which an orientation can be found in a graph such that it remains strongly connected. Authors in [4] propose several heuristics for the SNOP problem, while [2] addresses a bi-objective version of a similar problem, named Unidirectional Road Network design Problem (URND). We propose here an aggregated flow formulation (Section 2) and a column generation strategy (Section 3). Then, computational experiments and concluding remarks are given in Section 4.

2 Aggregated flow formulation

The aggregated multiflow formulation makes use of a decision variable $x_{ij} \in \{0, 1\}$ for each potential arc $(i, j) \in A$, stating if it is selected ($x_{ij} = 1$) or not ($x_{ij} = 0$). In order to ensure the strong connectivity, one unit of flow is sent from any node to any other node in V, using the flow variable $f^s_{ij} \geq 0$ for each origin node $s \in V$ and each arc $(i, j) \in A$. These flows also help computing the distance between s and the other nodes. Thus, the model is as follows:

$$\text{(Compact)} \min \sum_{(i,j)\in A} \sum_{s\in V} c_{ij} f^s_{ij} \qquad s.t. \tag{1}$$

$$x_{ij} + x_{ji} = 1 \quad \forall\ \{i, j\} \in E \tag{2}$$

$$\sum_{(s,i)\in A} f^s_{si} - \sum_{(i,s)\in A} f^s_{is} = n - 1 \; \forall s \in V \tag{3}$$

$$\sum_{(j,i)\in A} f^s_{ji} - \sum_{(i,j)\in A} f^s_{ij} = 1 \quad \forall s \in V, i \in V \setminus \{s\} \tag{4}$$

$$f^s_{ij} - (n - 1)x_{ij} \leq 0 \quad \forall s \in V, (i, j) \in A \tag{5}$$

$$x_{ij} \in \{0, 1\} \; \forall (i, j) \in A \tag{6}$$

$$f^s_{ij} \geq 0 \quad \forall s \in V, (i, j) \in A \tag{7}$$