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Clique cutsets beyond chordal graphs

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Abstract

Truemper configurations (thetas, pyramids, prisms, and wheels) have played an important role in the study of complex hereditary graph classes (e.g. the class of perfect graphs and the class of even-hole-free graphs), appearing both as excluded configurations, and as configurations around which graphs can be decomposed. In this paper, we study the structure of graphs that contain (as induced subgraphs) no Truemper configurations other than (possibly) universal wheels and twin wheels. We also study several subclasses of this class. We use our structural results to analyze the complexity of the recognition, maximum weight clique, maximum weight stable set, and optimal vertex coloring problems for these classes. We also obtain polynomial χ-bounding functions for these classes.

Keywords: clique, stable set, vertex coloring, structure, algorithms.

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1 Introduction

All graphs that we consider are finite, simple, and nonnull. We use standard terminology and notation. Given graphs G and H, we say that G is *H-free* if G does not contain (an isomorphic copy of) H as an induced subgraph. Given a family $\mathcal{H}$ of graphs, we say that a graph G is *$\mathcal{H}$-free* if G is H-free for all $H \in \mathcal{H}$. A class of graphs is *hereditary* if it is closed under induced subgraphs. A *hole* in a graph is an induced cycle of length at least four. A *chordal graph* is a graph that contains no holes.

Configurations known as thetas, pyramids, prisms, and wheels (defined below) have played an important role in the study of such diverse (and important) classes as the classes of regular matroids, balanceable matrices, perfect graphs, and even-hole-free graphs (for a survey, see [6]). These configurations are also called *Truemper configurations*, as they appear in a theorem due to Truemper [5] that characterizes graphs whose edges can be labeled so that all chordless cycles have prescribed parities.

A *theta* is any subdivision of the complete bipartite graph $K_{2,3}$. A *pyramid* is any subdivision of the complete graph K_4 in which one triangle remains unsubdivided, and of the remaining three edges, at least two edges are subdivided at least once. A *prism* is any subdivision of $\overline{C_6}$ in which the two triangles remain unsubdivided. A *3-path-configuration* (or *3PC* for short) is any theta, pyramid, or prism (see Fig. 1). A *wheel* (H, x) is a graph that consists of a hole H and a vertex x that has at least three neighbors in $V(H)$. A *universal wheel* is a wheel (H, x) such that x is adjacent to all vertices in $V(H)$. A *twin wheel* is a wheel (H, x) such that x has precisely three neighbors in $V(H)$, and those neighbors are consecutive vertices of H. A *proper wheel* is a wheel that is neither a universal wheel nor a twin wheel. A *cap* is a graph that consists of a chordless cycle of length at least four and a vertex adjacent to two consecutive vertices of the cycle (and to no other vertices of the cycle).

Here, we are interested in the hereditary classes $\mathcal{G}_{UT}$, $\mathcal{G}_U$, $\mathcal{G}_T$, and $\mathcal{G}_{UT}^{cap-free}$, defined as follows. $\mathcal{G}_{UT}$ is the class of all (3PC, proper wheel)-free graphs (so the only Truemper configurations that graphs in $\mathcal{G}_{UT}$ may contain are universal wheels and twin wheels); $\mathcal{G}_U$ is the class of all (3PC, proper wheel, twin wheel)-free graphs; $\mathcal{G}_T$ is the class of all (3PC, proper wheel, universal wheel)-free graphs; and $\mathcal{G}_{UT}^{cap-free}$ is the class of all (3PC, proper wheel, cap)-free graphs. Clearly, $\mathcal{G}_U$, $\mathcal{G}_T$, and $\mathcal{G}_{UT}^{cap-free}$ are proper subclasses of $\mathcal{G}_{UT}$; furthermore, the class of chordal graphs is a proper subclass of each of these four classes.

We first obtain decomposition theorems for the classes $\mathcal{G}_{UT}$, $\mathcal{G}_U$, $\mathcal{G}_T$, and $\mathcal{G}_{UT}^{cap-free}$, and then we use these theorems to analyze the complexity of the