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Maximum Cuts in Edge-colored Graphs [1]

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Abstract

The input of the Maximum Colored Cut problem consists of a graph $G = (V, E)$ with an edge-coloring $c : E \to \{1, 2, 3, \ldots, p\}$ and a positive integer $k > 0$, and the question is whether G has a nontrivial edge cut using at least k colors. The Colorful Cut problem has the same input but asks for a nontrivial edge cut using *all* colors. Unlike what happens for the classical Maximum Cut problem, we prove that both problems are NP-complete even on complete, planar, or bounded treewidth graphs. Furthermore, we prove that Colorful Cut is NP-complete even when each color class induces a clique of size at most 3, but is trivially solvable when each color induces a K_2. On the positive side, we prove that Maximum Colored Cut is fixed-parameter tractable when parameterized by either k or p, and that it admits a cubic kernel in both cases.

Keywords: colored cuts, edge cuts, max cut, planar graph, polynomial kernel.

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1 Introduction

Let $G = (V, E)$ be a simple graph with an edge coloring $c : E \to \{1, 2, \ldots, p\}$, not necessarily proper. Given a proper subset $S \subset V$, we define the *edge cut* ∂S as the subset of E where the edges have one endpoint in S and the other in $V \setminus S$. We represent by $c(\partial S)$ the set of colors that are in ∂S, i.e., $c(\partial S) = \{c(e) \mid e \in \partial S\}$. We are interested in the problem of finding a subset $S \subset V$ such that $|c(\partial S)| \geq |c(\partial T)|$ for every $T \subset V$. This problem is called Maximum Colored Cut and its decision form is stated next.

> Maximum Colored Cut
> INSTANCE: A graph $G = (V, E)$ with an edge coloring $c : E \to \{1, 2, \ldots, p\}$ and an integer $k > 0$.
> QUESTION: Is there a proper subset $S \subset V$ such that $|c(\partial S)| \geq k$?

The classical (simple) Maximum Cut problem [5] is the particular case of Maximum Colored Cut when $c : E \to \mathbb{N}$ is an injective function. Therefore, we are interested in analyzing the complexity of Maximum Colored Cut on graph classes $\mathcal{C}$ for which Maximum Cut is solvable in polynomial time. In addition, we are also interested in the complexity of determining if the input graph has a subset $S \subset V$ such that $|c(\partial S)| = p$, i.e., if there is an edge cut ∂S using all the colors; we call this problem Colorful Cut.

> Colorful Cut
> INSTANCE: A graph $G = (V, E)$ with an edge coloring $c : E \to \{1, 2, \ldots, p\}$.
> QUESTION: Is there a proper subset $S \subset V$ such that $c(\partial S) = p$?

Although the complexity of Minimum Colored Cut, which is defined analogously but with '$|c(\partial S)| \leq k$?', has been explored in recent years (cf., for instance, [2]), to the best of our knowledge a study on the complexity of Maximum Colored Cut is novel. As Colorful Cut is a particular case of Maximum Colored Cut, our hardness results deal with Colorful Cut while the tractable cases will be presented for Maximum Colored Cut.

2 NP-completeness

Hadlock [6] proved that (simple) Maximum Cut is polynomial-time solvable on planar graphs. In this section we prove, among other results, the NP-completeness of Colorful Cut on a particular subclass of planar graphs.