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Strong intractability of generalized convex recoloring problems [1]

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Abstract

A coloring of the vertices of a connected graph is r-convex if each color class induces a subgraph with at most r components. We address the r-convex recoloring problem defined as follows. Given a graph G and a coloring of its vertices, recolor a minimum number of vertices of G so that the resulting coloring is r-convex. This problem, known to be $\mathcal{NP}$-hard even on paths, was first investigated on trees and for $r = 1$, motivated by applications on perfect phylogenies. The concept of r-convexity, for $r \geq 2$, was proposed later, and it is also of interest in the study of protein-protein interaction networks and phylogenetic networks. Here, we show that, for each $r \in \mathbb{N}$, the r-convex recoloring problem on n-vertex bipartite graphs cannot be approximated within a factor of $n^{1-\varepsilon}$ for any $\varepsilon > 0$, unless $\mathcal{P} = \mathcal{NP}$. We also provide strong hardness results for weighted and parametrized versions of the problem.

Keywords: convex recoloring, hardness, inapproximability, parameterized intractability

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1 Introduction

A *total coloring* of a graph G is a function $C\colon V(G) \to \mathcal{C}$ that assigns to each vertex in $V(G)$ a color from $\mathcal{C}$. A *partial coloring* (or simply, a *coloring*) of G is a function $C : V(G) \to \mathcal{C} \cup \{\emptyset\}$, where a vertex v in G is *uncolored* when $C(v) = \emptyset$. We say that G is *k-colored* if its coloring uses k colors. For each color $c \in \mathcal{C}$, the *color class c* (denoted by $C^{-1}(c)$) is the set of vertices in G that have color c, that is, $C^{-1}(c) := \{v \in V(G)\colon C(v) = c\}$. Note that the coloring defined here differs from the classic vertex coloring, in which adjacent vertices are required to have different colors.

A *totally colored graph* is a pair (G, C) consisting of a graph G and a total coloring C of its vertices. We say that a total coloring C is *r-convex* if, for each $c \in \mathcal{C}$, the color class c induces a subgraph of G with at most r components. A (partially) *colored graph* is defined similarly. A *r-convex (partial) coloring* $C\colon V(G) \to \mathcal{C} \cup \{\emptyset\}$ is a (partial) coloring that can be extended to a r-convex total coloring by solely assigning a color in $\mathcal{C}$ to each uncolored vertex. Given a colored graph (G, C), any other coloring $C'\colon V(G) \to \mathcal{C} \cup \{\emptyset\}$ is a *recoloring* of (G, C), where $\mathcal{C}$ is the same set of colors than in the initial coloring C. We say that a vertex v is *recolored* (by C') if $C(v) \neq \emptyset$ and $C(v) \neq C'(v)$.

We focus here on the r-Convex Recoloring (r-CR) problem, defined as follows: *given a connected graph G, a coloring $C\colon V(G) \to \mathcal{C} \cup \{\emptyset\}$ and a weight function $w\colon V(G) \to \mathbb{Q}_{\geq 0}$, find an r-convex recoloring C' of (G, C) that minimizes the function $\sum_{v \in R_C(C')} w(v)$, where $R_C(C') := \{v \in V(G)\colon C(v) \neq \emptyset$ and $C(v) \neq C'(v)\}$ is the set of vertices recolored by C'.*

We start with some remarks on the r-CR problem. We may (and will) assume that the weight of any uncolored vertex is 0. If the function w only assigns weights 1 or 0 to the vertices of G, then we get a particular problem that we call r-CR-bin. More formally, r-CR-bin is the r-CR problem restricted to instances (G, C, w) such that, for each $v \in V(G)$, $w(v) = 1$ if $C(v) \neq \emptyset$; and $w(v) = 0$, otherwise. For simplicity, we denote by (G, C) an instance of this problem, which can be seen as a cardinality (or unweighted) version of r-CR.

The r-CR problem is of interest for applications related to protein-protein interaction networks and phylogenetic networks [5]. Additionally, when $r = 1$, r-CR is used in the contexts of routing problems, transportation networks [7] and phylogenetic trees [10].

The r-CR problem (with $r \geq 2$) was introduced by Chor et al. in 2007 [5]. In their work, they investigate a closely related problem called r-Connected Coloring Completion (r-CCC): *given a colored graph, decide whether it is possible to extend the initial coloring to a total r-convex coloring.* Chor et al. [5]