Available online at www.sciencedirect.com
ScienceDirect
Electronic Notes in DISCRETE MATHEMATICS
Electronic Notes in Discrete Mathematics 62 (2017) 99–104
www.elsevier.com/locate/endm

Intersection Graphs of Orthodox Paths in Trees

Claudson Ferreira Bornstein [1]

DCC-IM, Federal University of Rio de Janeiro, Rio de Janeiro, Brazil

José Wilson Coura Pinto [2]

PESC-COPPE, Federal University of Rio de Janeiro, Rio de Janeiro, Brazil

Dieter Rautenbach [3]

Institute of Optimization and Operations Research, Ulm University, Ulm, Germany

Abstract

We study the graph classes ORTH$[h, s, t]$ introduced by Jamison and Mulder, and focus on the case $s = 2$, which is closely related to the well-known VPT and EPT graphs. We collect general properties of the graphs in ORTH$[h, 2, t]$, and provide a characterization in terms of tree layouts. Answering a question posed by Golumbic, Lipshteyn, and Stern, we show that ORTH$[h+1, 2, t] \setminus$ ORTH$[h, 2, t]$ is non-empty for every $h \geq 3$ and $t \geq 3$. We derive decomposition properties, which lead to efficient recognition algorithms for the graphs in ORTH$[h, 2, 2]$ for every $h \geq 3$. Finally, we show that the graphs in ORTH$[3, 2, 3]$ are line graphs of planar graphs.

Keywords: Intersection graph, (h, s, t)-representation, orthodox (h, s, t)-representation, line graph, chordal graph

https://doi.org/10.1016/j.endm.2017.10.018
1571-0653/

1 Introduction

Intersection graphs are a well studied topic [12,8] and the intersection graphs of paths in trees have received special attention. In the present paper we study so-called orthodox representations with bounds on the maximum degree of the host tree as well as on the size of the intersections corresponding to adjacencies. The corresponding notions were introduced by Jamison and Mulder in [9,10,11].

For positive integers h, s, and t, an *(h, s, t)-representation* of a graph G is a pair $(T, \mathcal{S})$, where T is a tree of maximum degree at most h, and $\mathcal{S}$ is a collection $(S_u)_{u \in V(G)}$ of subtrees S_u of maximum degree at most s of T, one for each vertex u of G, such that two distinct vertices u and v of G are adjacent if and only if S_u and S_v have at least t vertices in common. An (h, s, t)-representation $(T, \mathcal{S})$ of G with $\mathcal{S} = (S_u)_{u \in V(G)}$ is *orthodox* if, for every vertex u of G, all leaves of S_u are also leaves of T, and, for every two distinct vertices u and v of G, the following three properties are equivalent:

(i) u and v are adjacent.

(ii) S_u and S_v have at least t vertices in common.

(iii) S_u and S_v share a leaf of T.

Let $[h, s, t]$ and ORTH$[h, s, t]$ be the hereditary classes of graphs that have an (h, s, t)-representation and an orthodox (h, s, t)-representation, respectively. If no upper bound on the maximum degree of the host T is imposed, we replace h with ∞. Similarly, if no upper bound on the maximum degree of the subtrees in $\mathcal{S}$ is imposed, we replace s with ∞.

Using this terminology, Gavril's famous result [3] states that the class of chordal graphs coincides with $[\infty, \infty, 1]$. Jamison and Mulder [10,11] attribute to McMorris and Scheinerman [13] the insight that

$$[\infty, \infty, 1] = \text{ORTH}[3, 3, 1] = \text{ORTH}[3, 3, 2].$$

The well studied vertex and edge intersection graphs of paths in trees (VPT-graphs and EPT-graphs) [4,5,6] coincide with $[\infty, 2, 1]$ and $[\infty, 2, 2]$, respectively. Golumbic and Jamison [5] have shown that deciding whether a given graph belongs to $[3, 2, 1]$ is NP-complete. Alcón, Gutierrez, and Mazzoleni [1] generalized this result for every $h \geq 3$. Golumbic, Lipshteyn, and Stern [7]

[1] Email: cfb@cos.ufrj.br

[2] Email: jwcoura@cos.ufrj.br

[3] Email: dieter.rautenbach@uni-ulm.de