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Facet-inducing inequalities and a cut-and-branch for the bandwidth coloring polytope based on the orientation model [1]

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Abstract

The *bandwidth coloring problem* (BCP) is a generalization of the well-known vertex coloring problem (VCP), asking colors to be assigned to vertices of a graph such that the absolute difference between the colors assigned to adjacent vertices is greater than or equal to a weight associated to the edge connecting them. In this work we present an integer programming formulation for BCP based on the orientation model for VCP. We present two families of valid inequalities for this formulation, show that they induce facets of the associated polytope, and report computational experience suggesting that these families are useful in practice.

Keywords: bandwidth coloring, integer programming, polyhedral combinatorics.

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1 Introduction

Let $G = (V, E)$ be a simple undirected graph and, for each edge $ij \in E$, let $d_{i,j} \in \mathbb{Z}_{\geq 0}$ be a weigth associated with ij. The *bandwidth coloring problem* (BCP) asks for a mapping $c : V \to \mathbb{Z}_{\geq 0}$ such that $|c(i) - c(j)| \geq d_{i,j}$ for each edge $ij \in E$, i.e., the absolute difference between colors assigned to adjacent vertices must be greater than or equal to the weight associated to the edge between them [2,7,9]. BCP is a direct generalization of the classical vertex coloring problem (VCP), which corresponds to the case $d_{i,j} = 1$ for every $ij \in E$.

A variety of methods have been proposed to deal with BCP, such as genetic algorithms [9], tabu search [7], local search [10], among others. Integer programming (IP) approaches have also been developed [4,6,8], all of them based on the *standard formulation* for VCP. In this work we propose a new IP formulation for BCP, based on a natural generalization of the *orientation model* [1] for VCP. We present both initial polyhedral and computational studies of this formulation.

2 Orientation model for BCP

For each vertex $i \in V$, we introduce the integer variable $x_i \in \mathbb{Z}_{\geq 0}$ representing the color assigned to i. For each edge $ij \in E$, $i < j$, we introduce the binary variable y_{ij} in such a way that $y_{ij} = 1$ if $x_i < x_j$, and $y_{ij} = 0$ otherwise. Finally, the variable $z_{\max}$ represents the maximum assigned color in the optimal solution. Let $\chi(G, d)$ be the minimum number of colors guaranteeing a feasible solution, and fix $s \geq \chi(G, d)$. In this setting, the proposed formulation is the following.

$$
\begin{array}{llll}
\text{Minimize} & z_{\max} & & (1)\\
\text{Subject to} & x_i + d_{i,j} \leq x_j + s(1 - y_{ij}) & \forall ij \in E,\ i < j & (2)\\
& x_j + d_{i,j} \leq x_i + s y_{ij} & \forall ij \in E,\ i < j & (3)\\
& z_{\max} \geq x_i & \forall i \in V & (4)\\
& x_i \in \mathbb{Z}_{\geq 0} & \forall i \in V & (5)\\
& y_{ij} \in \{0, 1\} & \forall ij \in E,\ i < j & (6)\\
& z_{\max} \in \mathbb{R} & & (7)
\end{array}
$$

In the above formulation, constraint (2) ensures that $y_{ij} = 0$ if $x_i > x_j$, for $ij \in E$. On the other hand, if $x_j > x_i$ then constraint (3) ensures that