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Structure and Interpretation of Dual-Feasible Functions

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Abstract

We study two techniques to obtain new families of classical and general Dual-Feasible Functions: a conversion from minimal Gomory–Johnson functions; and computer-based search using polyhedral computation and an automatic maximality and extremality test.

Keywords: integer programming, cutting planes, cut-generating functions, Dual-Feasible Functions, 2-slope theorem, computer-based search

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1 Introduction

The duality theory of integer linear optimization appears in several concrete forms. Inspired by the monograph [1], we study *(classical) Dual-Feasible Functions* (DFFs, cDFFs), which are defined as functions $\phi\colon D \to D$ such that $\sum_{i\in I} x_i \leq 1 \Rightarrow \sum_{i\in I} \phi(x_i) \leq 1$ for any family $\{x_i\} \subseteq D$ indexed by a finite index set I, where $D = [0, 1]$. In [1], these functions are studied alongside with *general DFFs* (gDFFs), which satisfy the same property for the extended domain $D = \mathbb{R}$.

DFFs appear to first have been studied by Lueker [7] to provide lower bounds for bin-packing problems. DFFs can derive feasible solutions to the dual problem of the LP relaxation efficiently, therefore providing fast lower bounds for the primal IP problem. The computation of bounds is also the main angle of exposition in the monograph [1]. Vanderbeck [9] studied the use of DFFs in several combinatorial optimization problems including the cutting stock problem, generating valid inequalities for these problems.

The *maximal* (pointwise non-dominated) DFFs are of particular interest since they provide better lower bounds and stronger valid inequalities. Maximality is not enough if the strongest bounds and inequalities are expected. A maximal DFF is said to be *extreme* if it cannot be written as a convex combination of two other maximal DFFs. Therefore, a hierarchy on the set of valid DFFs, which indicates the strength of the corresponding valid inequalities and lower bounds, has been defined [1]. This development is parallel to the one in the study of cut-generating functions [10], to which there is a close relation that deserves to be explored in greater depth. Indeed, the characterization of minimal cut-generating functions in the Yıldız–Cornuéjols model [10] can be easily adapted to give a full characterization of maximal general DFFs, which is missing in [1].

The authors of [1] study analytical properties of extreme DFFs and use them to prove the extremality of various classes of functions, most of which are piecewise linear (possibly discontinuous).

In our paper, we complement this study by transferring recent algorithmic techniques [2,5] developed by Basu, Hildebrand, Hong, Köppe, and Zhou for cut-generating functions in the Gomory–Johnson model [3] to DFFs. In our software, available as the feature branch `dual_feasible_functions` in [5], we implement an automatic maximality and extremality test for classical DFFs.

In our software, written in SageMath [8], a comprehensive Python-based open source computer algebra system, we also provide an electronic compendium of the known extreme DFFs from [1]. We hope that it facilitates