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Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 62 (2017) 171–176 www.elsevier.com/locate/endm

Weighted upper domination number

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Abstract

The cardinality of a maximum minimal dominating set of a graph is called its upper domination number. The problem of computing this number is generally **NP**-hard but can be solved in polynomial time in some restricted graph classes. In this work, we consider the complexity and approximability of the weighted version of the problem in two special graph classes: planar bipartite, split. We also provide an inapproximability result for an unweighted version of this problem in regular graphs.

Keywords: Maximum weighted minimal dominating set (WUDS); **NP**-hard; inapproximability; planar bipartite; split graphs; UDS in regular graphs.

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https://doi.org/10.1016/j.endm.2017.10.030

1571-0653/

1 Introduction

Given a simple graph $G = (V, E)$, a *dominating set* $D \subseteq V$ is a subset of vertices such that $\forall v \notin D$, $\exists u \in D$ with $[u, v] \in E$. D is said to be a *minimal dominating set* of G if D is minimal for inclusion, i.e., $\forall v \in D$, $D - v$ is not a dominating set. For $x \in V$, $N_G(x)$ denotes the neighborhood of x while $N_G[x]$ denotes the closed neighborhood of x, that is $N_G[x] = N_G(x) \cup \{x\}$. For $A \subseteq V$, $N_G(A) = \cup_{x \in A} N_G(x)$ and $N_G[A] = \cup_{x \in A} N_G[x]$. For a dominating set $D \subseteq V$ and a vertex $x \in D$, let $s_D(x) = N_G(x) - N_G[D - x]$ and $s_D[x] = N_G[x] - N_G[D - x]$. The set $s_D(x)$ (resp., $s_D[x]$) corresponds to the private vertices of $V - D$ (resp., $V - D \cup \{x\}$) that are only dominated by x. In other words, $s_D(x) = \{y \notin D : N_G(y) \cap D = \{x\}\}$. It is well known that D is a minimal dominating set of G iff $N_G[D] = V$ and $\forall x \in D$, $s_D[x] \neq \emptyset$. Here, we consider a simple graph $G = (V, E)$ where each node $v \in V$ is weighted by $w(v) \geq 0$, a non-negative integer. For $U \subseteq V$, $w(U) = \sum_{v \in U} w(v)$.

Definition 1.1 The Weighted Upper dominating set problem (WUDS for short) consists in finding, given a simple node weighted graph $G = (V, E, w)$, a minimal dominating set U of G with maximum weight $w(U)$. The weight of any optimal solution of G will be denoted by $\Gamma_w(G)$ and it will be called the weighted upper domination number of G.

2 Related papers

To our best knowledge, the complexity of computing the weighted upper domination number has never been studied in the literature, while several results appear for the unweighed case (corresponding to $w(v) = 1$ for every $v \in V$). In this case, the size of any maximum minimal dominating set of G is usually denoted by $\Gamma(G)$ and it is known in the literature as *the upper domination number* of G. The complexity of computing $\Gamma(G)$ has already been studied for the main classes of graphs. For instance, it was shown to be **NP**-hard for general graphs in [6] and **W[2]**-hard in [3], for graphs of maximum degree 3 and planar graphs in [4], co-bipartite graphs in [1]. A dichotomy result for monogenic classes of graphs (i.e. classes defined by a single forbidden induced subgraph) is also given in [1], where it is proved that if the only forbidden induced subgraph is a P_4 or a $2K_2$ (or any induced subgraph of these graphs), then computing $\Gamma(G)$ can be solved in polynomial time; otherwise, it is **NP**-hard. The first boundary property for upper domination numbers was given very recently in [2]. The upper domination number is one of the most inapproximable **NPO** problems because it has been proved in [4] that for any