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k-quasi-transitive digraphs of large diameter

César Hernández-Cruz [1]

Instituto de Matemáticas
Universidad Nacional Autónoma de México
Ciudad de México, México

Jesús Alva-Samos [2]

Instituto de Matemáticas
Universidad Nacional Autónoma de México
Ciudad de México, México

Abstract

Let k be an integer, $k \geq 2$. A digraph $D = (V, A)$ is k-quasi-transitive if for every pair of vertices $u, v \in V$, the existence of a directed path of length k from u to v implies the existence of the arc (u, v) or (v, u) in $A(D)$. Under this definition, quasi-transitive digraphs are 2-quasi-transitive digraphs.

A recursive characterization (the so-called Canonical Decomposition) is known for quasi-transitive digraphs, but no characterization is known for k-quasi-transitive digraphs in the general case. Recently, Wang and Zhang proved that if k is an even integer, then a k-quasi-transitive digraph of diameter at least $k + 2$ admits a partition of its vertex set into two parts, each of them inducing a semicomplete digraph. In this work, we will present an analogous result for the case when k is an odd integer and discuss some of its consequences and future lines of research.

Keywords: quasi-transitive digraph, k-quasi-transitive digraph, traceability, Hamiltonicity

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1 Introduction

When dealing with a difficult problem in graph theory, it is customary to consider the restriction of the problem to a well-understood family of graphs; this way, it is often possible to gain an insight in the complexities of the problem. Sometimes the problem remains hard to attack even for a relatively simple family of graphs, e.g., the problem of determining if a graph is Hamiltonian is $\mathcal{NP}$-complete even when restricted to the family of split graphs. For undirected graphs, many families have been widely studied, characterized, and are deeply understood. Nonetheless, there are only a handful of digraph families that have been extensively studied. One such family is the class of quasi-transitive digraphs, which is a common generalization of both tournaments and transitive digraphs. In this paper we will study k-quasi-transitive digraphs, a natural generalization of quasi-transitive digraphs, and show that, under certain circumstances (large diameter), they have a very nice structure. Our results complement those of Wang and Zhang [11], and give us a much better understanding of this class of digraphs, proving it to be interesting.

All digraphs considered in this work are finite, without loops, and without parallel arcs, but symmetric arcs are allowed. All walks (including paths and cycles) will be considered to be directed, unless otherwise specified. For basic undefined terms we refer the reader to [2]. In particular, if $D = (V, A)$ is a digraph, and $x, y \in V$, we will write $x \to y$ instead of $(x, y) \in A$; by $\overline{xy}$ we will mean that $x \to y$, or $y \to x$, or both. An (x, y)-walk is a walk with initial vertex x and terminal vertex y. The digraph D is semicomplete if for every pair of vertices $x, y \in V$, we have $\overline{xy}$.

A digraph D is *k-quasi-transitive* (*k-transitive*) if for every pair of vertices x, y of D, the existence of an (x, y)-path of length k, implies $\overline{xy}$ ($x \to y$). A *quasi-transitive* (*transitive*) digraph is a 2-quasi-transitive (2-transitive) digraph. The class of k-quasi-transitive digraphs was introduced by Galeana-Sánchez and Hernández-Cruz in [8], where they proved the first general structural properties for this family. Initially, quasi-transitive digraphs were studied by Ghouila-Houri in [6] because of their relation to comparability graphs[3]. Nonetheless, in their seminal paper of 1995 [1], Bang-Jensen and Huang began the study of this family in its own right. Through the last 20 years, quasi-transitive digraphs have gained a place among the most studied and better

[1] Email: cesar@matem.unam.mx

[2] Email: jesusalvasamos@ciencias.unam.mx

[3] He proved that a graph G admits a quasi-transitive orientation if and only if it admits a transitive orientation if and only if it is a comparability graph