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On Generalizations of the Parking Permit Problem and Network Leasing Problems

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Abstract

We propose a variant of the parking permit problem, called multi parking permit problem, in which an arbitrary demand is given at each instant and one may buy multiple permits to serve it. We show how to reduce this problem to the parking permit problem, while losing a constant cost factor. We obtain a 4-approximation algorithm and, for the online setting, a deterministic $O(K)$-competitive algorithm and a randomized $O(\lg K)$-competitive algorithm, where K is the number of permit types. For a leasing variant of the Steiner network problem, these results imply $O(\lg n)$-approximation and online $O(\lg K \lg |V|)$-competitive algorithms, where n is the number of requests and $|V|$ is the size of the input metric. Also, our technique turns into polynomial-time the pseudo-polynomial algorithms by Hu, Ludwig, Richa and Schmid for the 2D parking permit problem. For a leasing variant of the buy-at-bulk network design problem, these results imply: (i) an algorithm which improves the best previous approximation, and (ii) the first competitive online algorithm.

Keywords: leasing optimization, Steiner network, buy-at-bulk network design, approximation algorithms, competitive online algorithms.

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1 Introduction

In the **parking permit problem** (PP), proposed by Meyerson [10], we have K types of permits with lengths $\delta_1, \ldots, \delta_K$ and costs $\gamma_1, \ldots, \gamma_K$, and we are given a sequence $r_0, \ldots, r_{T-1} \in \{0, 1\}$. The goal is to find a minimum-cost set $S \subseteq [K] \times \{0, \ldots, T-1\}$ of permits such that, for each t with $r_t = 1$, we have some permit $(k, \hat{t}) \in S$ satisfying $t \in [\hat{t}, \hat{t} + \delta_k)$. PP has a polynomial exact dynamic programming algorithm. In the online version, T is unknown and $r_0, \ldots, r_{T-1}$ are revealed one at a time, and the problem has deterministic $\mathrm{O}(K)$-competitive algorithm and $\Omega(K)$ lower bound, as well as randomized $\mathrm{O}(\lg K)$-competitive algorithm and $\Omega(\lg K)$ lower bound [10]. We assume

$$1 = \delta_1 < \delta_2 < \ldots < \delta_K \text{ and } \gamma_k/\delta_k < \gamma_\ell/\delta_\ell \text{ for } k > \ell. \tag{1}$$

PP is the seminal problem of the **leasing optimization model**, in which each resource may be leased for different periods of time, and it is more cost-effective to lease resources for longer periods. This contrasts with traditional optimization models, in which acquired resources last for unlimited duration. Leasing optimization may be applied to both offline and online problems. Some literature has been devoted to leasing variants of optimization problems [2,11,1]. Also, variants of PP and related problems were studied [9,8,6]. A traditional problem whose leasing variant was studied by Meyerson [10] is the Steiner forest problem. In the **Steiner leasing problem** (SLe), each edge can be leased for finite periods of time. Meyerson presented a relationship between SLe and PP: if the input metric is a tree, SLe reduces to solve PP for each edge. Using the technique of approximating a metric by a tree metric [4,5], a solution for a generic input can be obtained, losing some guarantee of quality. This idea can be formalized as follows.

Theorem 1.1 ([4,5]) *Given a minimization problem on a finite metric (V, d) whose objective function is a non-negative linear combination of distances in d, if there is an α-competitive algorithm for the special case of tree metrics, then there is a randomized $\mathrm{O}(\alpha \lg |V|)$-competitive algorithm for the general case.*

In this paper, we extend this approach to solve leasing variants of two network design problems. In the **Steiner network problem** (SN) [7,12], we are given pairs of vertices and a demand $r(u, v)$ for each pair (u, v), and we wish to buy a minimum-cost multiset of edges that contains $r(u, v)$ edge-disjoints (u, v)-paths, for each pair (u, v). In the **buy-at-bulk network design problem** (BaBND) [3], we also have pairs of vertices with demands, but we can install cables on each edge with different capacities per length, and we wish