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# A linear-time algorithm for the identifying code problem on block graphs [1]

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## Abstract

The identifying code problem is a special search problem, challenging both from a theoretical and from a computational point of view, even for several graphs where other usually hard problems are easy to solve, like bipartite graphs or chordal graphs. Hence, a typical line of attack for this problem is to determine minimum identifying codes of special graphs. In this work we study the problem of determining the cardinality of a minimum identifying code in block graphs (that are diamond-free chordal graphs). We present a linear-time algorithm for this problem, as a generalization of a linear-time algorithm proposed by Auger in 2010 for the case of trees. Thereby, we provide a subclass of chordal graphs for which the identifying code problem can be solved in linear time.

*Keywords:* identifying codes, block graphs, computational complexity

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# 1 Introduction

Many search problems as, e.g., fault detection in networks or fire detection in buildings, can be modeled by so-called identifying codes in graphs [10].

Consider a graph $G = (V, E)$ and denote by $N[i] = \{i\} \cup N(i)$ the closed neighborhood of a vertex $i$. A subset $C \subseteq V$ is *dominating* (resp. *identifying*) if $N[i] \cap C$ are non-empty (resp. distinct) sets for all $i \in V$. An *identifying code* of $G$ is a vertex subset which is dominating and identifying.

Not every graph $G$ admits an identifying code, i.e. is *identifiable*: this holds if and only if there are no true twins in $G$, i.e., there is no pair of distinct vertices $i, j \in V$ with $N[i] = N[j]$ [10]. On the other hand, the whole vertex set of every identifiable graph trivially forms an identifying code.

The *identifying code number* $\gamma_{ID}(G)$ of a graph $G$ is the minimum cardinality of an identifying code of $G$. Determining $\gamma_{ID}(G)$ is in general NP-hard [6] and remains hard for several graph classes where other in general hard problems are easy to solve, including bipartite graphs [6] and two classes of chordal graphs, namely split graphs and interval graphs [7].

The identifying code problem has been actively studied during the last decade, where typical lines of attack are to determine minimum identifying codes of special graphs or to provide bounds on their size. Closed formulas for the exact value of $\gamma_{ID}(G)$ have been found so far only for restricted graph families (e.g. for paths and cycles [5], for stars [8], for complete multipartite graphs [1] and some subclasses of split graphs [2]).

A linear-time algorithm to determine $\gamma_{ID}(G)$ if $G$ is a tree was provided by Auger [3]. In this paper, we determine the identifying code number of block graphs (that are diamond-free chordal graphs [4]). We present a linear-time algorithm for this problem, as a generalization of the linear-time algorithm by Auger for trees. Thereby, we provide a subclass of chordal graphs for which the identifying code problem can be solved in linear time.

A *block graph* is a graph in which every maximal 2-connected subgraph (block) is a clique (see Fig. 1). Block graphs are precisely those chordal graphs in which every two maximal cliques have at most one vertex in common [9]. Note that a block graph $B$ is identifiable (i.e. has no true twins) if and only if each maximal clique $K$ of $B$ satisfies that all vertices in $K$, except at most one, have a neighbor that is not in $V(K)$. Moreover, if we call $V(K')$ such neighbors then $C$ is an identifying code of $B$ if and only if $V(K') \subset C$.