Available online at www.sciencedirect.com

ScienceDirect

Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 62 (2017) 267–272 www.elsevier.com/locate/endm

Bounds on Directed star arboricity in some digraph classes

Henri Perret du Cray [1] Mourad Baïou [2] Laurent Beaudou [3]
Vincent Limouzy [4]

LIMOS, Université Clermont Auvergne, Aubière, France

Abstract

A *galaxy* is a forest of directed stars. The notion of galaxy can be related to Facility Location problems as well as wavelength assignment problems in optical networks. Amini et al. [2] and Gonçalves et al. [6] gave bounds on the minimum number of galaxies needed to cover the arcs of a digraph D, called directed star arboricity ($dst(D)$). They conjectured that those bounds could be improved such that $dst(D) \leq \Delta(D)$, for $\Delta(D) \geq 3$ and $dst(D) \leq 2\Delta^+(D)$ for $\Delta^+(D) \geq 2$. In this work, we study the directed star arboricity in two non-trivial digraph classes: k-degenerate digraphs and tournaments.

Keywords: directed star arboricity, galaxy, digraph, tournament

1 Introduction

We consider simple digraphs without loops (multiple arcs are not allowed). Given a digraph $D = (V, A)$ and a vertex v of D, $d^+(v)$ (resp. $d^-(v)$) is the

[1] Email: henri.perret_du_cray@uca.fr
[2] Email: mourad.baiou@uca.fr
[3] Email: laurent.beaudou@uca.fr
[4] Email: vincent.limouzy@uca.fr

https://doi.org/10.1016/j.endm.2017.10.046

out-degree (resp. in-degree) of v. Let Δ^+ (resp. Δ^-) denote the maximum out-degree (resp. in-degree) of a digraph D, or $\Delta^+(D)$ (resp. $\Delta^-(D)$) if there is some ambiguity about the digraph D. Let us denote $d(v) = d^+(v) + d^-(v)$ the degree of v in D and denote $\Delta(D) = \max\{d(v)|v \in V(D)\}$ the maximum degree of a digraph D.

In the following, n is the number of vertices of D. We say that a digraph is *even* (resp. *odd*) if n is even (resp. odd). Given a vertex v of D, let $N^+(v)$ (resp. $N^-(v)$) be the out-neighborhood (resp. in-neighborhood) of v and $D\backslash v$ the digraph obtained by removing v from D.

A *directed star* is a digraph with at least two vertices such that one vertex has out-degree zero and other vertices have in-degree zero and out-degree one (see Fig. 1). A *galaxy* is a set of vertex-disjoint stars.

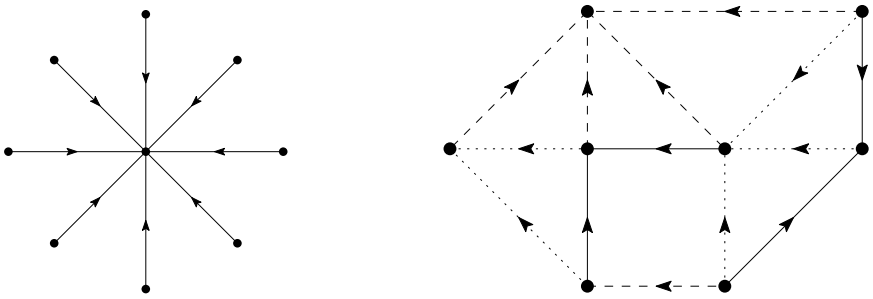

Fig. 1. A directed star and a directed star coloring.

The *directed star k-coloring* problem is that of deciding whether or not there exists a partition of the arcs of a digraph D into k galaxies. In the rest of this work the word *coloring* will refer to this specific coloring of the arcs of D. This decision problem is NP-complete even when restricted to directed acyclic graphs without path of length three [3].

The *directed star arboricity* $dst(D)$ of a digraph D, a notion introduced by Algor and Alon [1], is the minimum number of galaxies needed to cover all the arcs of D. In other words, the directed star arboricity is the number of colors needed to partition the arcs into galaxies. This parameter is related to wavelength assignment in star networks [5], and can also be connected to facility location problems in combinatorial optimization [3].

In [2] and [6], the authors bounded the directed star arboricity. They showed that for any digraph D, $\Delta + 1$ or $2\Delta^+ + 1$ galaxies are sufficient to cover the arcs of D. Amini et al. [2] conjectured that these bounds could be improved as follows:

(1) Every digraph D with $\Delta^+ \geq 2$ satisfies $dst(D) \leq 2\Delta^+$.