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The minimum chromatic violation problem: a polyhedral study [1]

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Abstract

We propose a generalization of the k-coloring problem, namely the *minimum chromatic violation problem* (MCVP). Given a graph $G = (V, E)$, a set of *weak edges* $F \subset E$ and a set of colors $\mathcal{C}$, the MCVP asks for a $|\mathcal{C}|$-coloring of the graph $G' = (V, E \backslash F)$ minimizing the number of weak edges with both endpoints receiving the same color. We present an integer programming formulation for this problem and provide an initial polyhedral study of the polytopes arising from this formulation. We give partial characterizations of facet-inducing inequalities and we show how facets from weaker and stronger instances of MCVP (i.e., more/less weak edges) are related. We then introduce a general lifting procedure which generates (sometimes facet-inducing) valid inequalities from generic valid inequalities and we present several facet-inducing families arising from this procedure. Finally, we present another family of facet-inducing inequalities which is not obtained from the prior lifting procedure.

Keywords: Vertex coloring, Integer programming, Chromatic violation.

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1 Introduction

A k-coloring of a graph is a partition of the vertex set in k stable sets (i.e., sets of pairwise non-adjacent vertices). The k-coloring problem asks whether a given graph has a k-coloring or not, and it is known to be NP-Complete if $k \geq 3$. The classical vertex coloring problem (VCP) asks for the smallest k needed to color a given graph and it has many known applications such as frequency assignment problems, course timetabling, scheduling problems, etc. In practice, it is not difficult to find situations where the value for k is actually fixed and the goal is to minimize a *conflict-like* notion among some vertices in the same color class. A straightforward example of this is a minimum interference frequency k-assignment. In order to address this kind of problems, we propose a generalization of the k-coloring problem, namely the *minimum chromatic violation problem* (MCVP), which considers a graph $G = (V, E)$, a set of colors $\mathcal{C}$ and a subset of *weak* edges $F \subset E$, and asks for a $|\mathcal{C}|$-coloring of $G' = (V, E \backslash F)$ minimizing the number of edges from F with both endpoints in the same color class. When $F = \emptyset$ then MCVP is the k-coloring problem, hence MCVP is NP-Hard. Moreover, MCVP also generalizes the k-partition problem, when $F = E$ [3]. Although there have been some polyhedral approaches for the k-partition problem, we found significant differences between these polytopes and the general case of MCVP. Since integer programming (IP) techniques have shown to be quite successful for VCP we propose to tackle MCVP with such techniques. In this paper we present an IP formulation for MCVP and provide an initial polyhedral study of it. We give partial characterizations of facet-inducing inequalities and we show how facets from weaker and stronger instances of MCVP (i.e., more/less weak edges) are related. We then introduce a general lifting procedure which generates (sometimes facet-inducing) valid inequalities from generic valid inequalities and we present several facet-inducing families arising from this procedure. Finally, we present another family of facet-inducing inequalities which is not obtained from the prior lifting procedure. All along the paper, we call $G = (V, E)$ the input graph, $F \subset E$ the set of *weak* edges, and $\mathcal{C}$ the set of available colors in an MCVP instance. We also say that an edge is a *strong* edge if it belongs to $E \backslash F$ and we write $G - E'$ as a shortcut for $G' = (V, E \backslash E')$, with $E' \subset E$.

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