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Transfinite mutations in the completed infinity-gon

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We introduce mutation along infinite admissible sequences for infinitely marked surfaces, that is surfaces with infinitely many marked points on the boundary. We show that mutation along such admissible sequences produces a preorder on the set of triangulations of a fixed infinitely marked surface. We provide a complete classification of the strong mutation equivalence classes of triangulations of the infinity-gon and the completed infinity-gon respectively, where strong mutation equivalence is the equivalence relation induced by this preorder. Finally, we introduce the notion of transfinite mutations in the completed infinity-gon and show that all its triangulations are transfinitely mutation equivalent, that is we can reach any triangulation of the completed infinity-gon from any other triangulation via a transfinite mutation.

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0. Introduction

Triangulations of surfaces with marked points give rise to an interesting class of cluster algebras, which are tractable but sufficiently complicated to display a rich array of cluster combinatorics. The fact that they come together with a natural topological model means they play a key role in advancing our understanding of cluster theory, serving as important examples to test theories about general cluster algebras and categories (cf. for example [3] [5], [9] [12] and [13]). Traditionally, only triangulations of surfaces with finitely many marked points have been studied in the context of cluster theory. With the rising interest in cluster algebras and categories of infinite rank (cf. for example [6], [7], [8] [9], [11]), it is natural to extend the theory to a setting with infinitely many marked points, and consider what we call *infinitely marked surfaces*.

The idea to consider triangulations and mutations of infinitely marked surfaces is not new and has been executed in the context of cluster categories for example in [9] and [11] and in the context of cluster algebras in [6] and [7]. By introducing infinitely many marked points, interesting phenomena occur which do not appear in the finite setting. One notable feature of infinitely marked surfaces, as opposed to finitely marked surfaces, is that two different triangulations are in general not connected by finitely many mutations. In particular, two distinct triangulations of the same infinitely marked surface will in general give rise to two distinct cluster algebras of infinite rank in the sense of [6].

In the present paper we study infinite mutations for infinitely marked surfaces, motivated by overcoming the finiteness constraints of the classical theory. We introduce the notion of mutation along infinite admissible sequences, and show such mutations connect previously disconnected components of the exchange graph. In fact, examples of mutations in cluster algebras along infinite admissible sequences have previously been used in [8], and we formalize the idea here in the context of infinitely marked surfaces. We consider two important examples in more detail: the ∞-gon, which can be pictured as the line of integers, and the completed ∞-gon, which we obtain from the ∞-gon by completing with points at $\pm\infty$. Our main reason for studying these examples is that they provide combinatorial models for relatively well-studied examples in cluster theory of infinite rank: the ∞-gon relates to the cluster category studied in [9], and in more generality in [11], and to the cluster algebras studied in [6], whereas the completed ∞-gon relates to the representation theory of a polynomial ring in one variable.

Single mutations are involutive, and therefore, if we can mutate from a triangulation T to a triangulation T' in finitely many steps, there is a way to mutate back from T' to T. This is not the case anymore if we consider mutation along infinite admissible sequences. In this sense, we can think of mutations along infinite admissible sequences as