Journal of Combinatorial Theory, Series A 154 (2018) 598–609

Contents lists available at ScienceDirect

Journal of Combinatorial Theory, Series A

www.elsevier.com/locate/jcta

Some extremal results on complete degenerate hypergraphs

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ARTICLE INFO

Article history:
Received 30 December 2016

Keywords:
Turán number
Degenerate hypergraph
Random algebraic method
Alon–Shikhelman

ABSTRACT

Let $K^{(r)}_{s_1,s_2,\cdots,s_r}$ be the complete r-partite r-uniform hypergraph and $ex(n, K^{(r)}_{s_1,s_2,\cdots,s_r})$ be the maximum number of edges in any n-vertex $K^{(r)}_{s_1,s_2,\cdots,s_r}$-free r-uniform hypergraph. It is well-known in the graph case [19,18] that $ex(n, K_{s,t}) = \Theta(n^{2-1/s})$ when t is sufficiently larger than s. In this note, we generalize the above to hypergraphs by showing that if s_r is sufficiently larger than $s_1, s_2, \cdots, s_{r-1}$ then

$$ex(n, K^{(r)}_{s_1,s_2,\cdots,s_r}) = \Theta\left(n^{r-\frac{1}{s_1 s_2 \cdots s_{r-1}}}\right).$$

This follows from a more general Turán type result we establish in hypergraphs, which also improves and generalizes some recent results of Alon and Shikhelman [2]. The lower bounds of our results are obtained by the powerful random algebraic method of Bukh [6]. Another new, perhaps unsurprising insight which we provide here is that one can also use the random algebraic method to construct non-degenerate (hyper-)graphs for various Turán type problems.

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[1] Research partially supported by NSFC grants 11501539 and 11622110.

https://doi.org/10.1016/j.jcta.2017.10.002
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The asymptotics for $ex(n, K^{(r)}_{s_1,s_2,\cdots,s_r})$ is also proved by Verstraëte [25] independently with a different approach.
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1. Introduction

Let $r \geq 2$ be an integer and H, F be two r-uniform hypergraphs. An r-uniform hypergraph is called *F-free* if it contains no copy of F as its subhypergraph. For any integer n, let $ex(n, H, F)$ be the maximum number of copies of H in an n-vertex F-free r-uniform hypergraph. In case H is a single edge, this function then converts to the *Turán number* $ex(n, F)$ of the hypergraph F.

The study of Turán numbers $ex(n, F)$ is the main focus of the extremal graph theory. This was initiated by the celebrated theorems of Mantel [21] and Turán [24], which determine the precise value of $ex(n, F)$ when F is a complete graph. For general graphs F, Erdős–Stone and Simonovits [13,12,23] resolved Turán numbers $ex(n, F)$ asymptotically, except for bipartite graphs F (which are often called the *degenerate graphs*). Even to date there are still few degenerate graphs the asymptotics of whose Turán numbers are known. One of such examples is the complete bipartite graph $K_{s,t}$. A well-known theorem of Kővári, Sós and Turán [19] shows that $ex(n, K_{s,t}) = O(n^{2-1/s})$ for any integers $t \geq s$. For $s = 2, 3$, matched lower bounds were found in [11,5] respectively; for other values of s, this bound was known to be tight when t is sufficiently larger than s, which was first proved by Kollár, Rónyai and Szabó [18] and then slightly improved to $t > (s-1)!$ by Alon, Rónyai and Szabó [1]. Recently, Blagojević, Bukh and Karasev [3] and Bukh [6] used the random algebraic method to give different constructions which yield the same lower bound $ex(n, K_{s,t}) = \Omega(n^{2-1/s})$ as in [18,1], provided that t is sufficiently large. For more extremal results on generate graphs, we refer to the survey [14].

For the function $ex(n, H, F)$ where H, F are graphs and H is not an edge, there are only sporadic results such as [9,4,16] in the literature, until recently Alon and Shikhelman [2] systematically investigate this general function and obtain a number of results on complete graphs, complete bipartite graphs and trees. Among other results, they proved in [2] that for $s \geq 2m-2$ and $t \geq (s-1)! + 1$,

$$ex(n, K_m, K_{s,t}) = \Theta\left(n^{m - \frac{m(m-1)}{2s}}\right) \tag{1}$$

and for $(a-1)! + 1 \leq b < (s+1)/2$ and $t \geq s$,

$$ex(n, K_{a,b}, K_{s,t}) = \Theta\left(n^{a+b-ab/s}\right). \tag{2}$$

By contrast with the graph case, very little was known for the hypergraph Turán problems. (For instance, it is not known the value of $ex(n, F)$ when F is a complete

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