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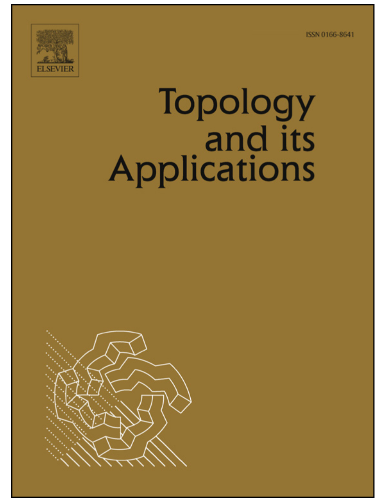

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Abstract

In this paper, we prove that a 'punctured' closed, connected, orientable triangulated manifold is simple homotopy equivalent to any of its roots. We also emphasize that this phenomena does not hold in general. Orientability plays a central role for this result and thus makes the result interesting. In the course of the proof of this theorem, we prove two lemmas, which partially answer two questions of Olivier Bernardi and Caroline Klivans.

Keywords: simplicial complex, cycle, forest, discrete morse theory, homology
MSC code: 05C05, 05C50, 05E45, 57Q10

1. Introduction

The notion of forests and their roots in higher dimensions was introduced by Olivier Bernardi and Caroline Klivans in [1] to obtain a generalization of the "Matrix Forest Theorem" in higher dimension. In fact, they proved that, for a $d-$dimensional simplicial complex G,

$$\sum_{(F,R)} |H_{d-1}(F,R)|^2 x^{|R|} = det(L_G + xId),$$

where the summation runs over all rooted forests of G and Id is the identity matrix of dimension $|G_{d-1}|$, and L_G is the Laplacian matrix. These higher dimensional forests and roots(definitions later) are direct generalization of the same in graphs. Now, in graph case, a forest simplicially collapses onto any of its roots (in graph case, a root consists of a finite number of vertices, one vertex per connected component of the graph). So it is tempting to