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General properties of pseudo-contractibility

Félix Capulín [a,*], Leonardo Juárez-Villa [a,*], Fernando Orozco-Zitli [a,*]

[a] *Universidad Autónoma del Estado de México, Facultad de Ciencias, Instituto Literario No. 100, Col. Centro, C.P. 50000, Toluca, Estado de México, Mexico*

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ABSTRACT

General facts about pseudo-homotopies and pseudo-contractibility are studied for topological spaces and continua. As a consequence of these, we find several conditions that obstruct pseudo-contractibility and we present examples of pseudo-contractible continua and non-pseudo-contractible continua.

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1. Introduction

The concept of pseudo-contractibility was introduced by R. H. Bing. However, W. Kuperberg gave the first example which proves that the notions of pseudo-contractibility and contractibility are different. This example was never published by himself but it is known among continuum theorists. He also asked whether or not the space $\sin\left(\frac{1}{x}\right)$ curve is pseudo-contractible (see [12]). H. Katsuura proves in [8] that the space $\sin\left(\frac{1}{x}\right)$ curve is not pseudo-contractible with factor space itself. In the same paper he proves that if the factor space Y is a nondegenerate indecomposable continuum such that each one of their composants is arc-wise connected, and if X is a continuum having a proper nondegenerate arc component, then X is

* Corresponding authors.
E-mail addresses: fcapulin@gmail.com (F. Capulín), juvile06@gmail.com, ljuarezv@uaemex.mx (L. Juárez-Villa), forozcozitli@gmail.com (F. Orozco-Zitli).

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not pseudo-contractible with factor space Y. After that, W. Dębski proves in [5] that the space $\sin(\frac{1}{x})$ curve is not pseudo-contractible. On the other hand, M. Sobolewski in [17] shows that the only (up homeomorphism) pseudo-contractible chainable continuum is the arc. This shows that the pseudo-arc is not pseudo-contractible, answering Problem 118 of [12]. The interested reader is referred to [1], [7], [8], [12] and [17] for getting more information about these results.

This paper is divided in nine sections. After preliminaries, we give, in sections three and four, several and general facts about pseudo-homotopies and pseudo-contractibility. In section five, pseudo-contractibility with respect to a topological space is studied. The concept pseudo-homotopy equivalent is related with pseudo-contractibility in section six. In sections seven and eight we give conditions which imply nonpseudo-contractibility. Finally in section nine we present some open questions about it.

2. Preliminaries

A *continuum* means a nonempty compact connected metric space. A topological space is said to be *continuumwise connected* provided that any two of its points are contained in a proper subcontinuum of the space. A *mapping* means a continuous function. Let X and Y topological spaces, we write $X \approx Y$ if X is homeomorphic to Y. An *arc* is understood as a homeomorphic image of the closed unit interval $I = [0, 1]$. If any two points of a space can be joined by an arc lying in the space, then the space is said to be *arcwise-connected*.

Let X and Y be topological spaces. The symbol $C(X, Y)$ denotes the topological space of all mappings from X to Y endowed with the compact-open topology. It is well known that if X is compact and Y is a compact metric space, then the compact-open topology coincides with the topology given by the supremum metric on $C(X, Y)$.

Let (X_1, τ_1) and (X_2, τ_2) be topological spaces such that $X_1 \cap X_2 = \emptyset$. The *free union* of X_1 and X_2 is the topological space (X, τ), where $X = X_1 \cup X_2$ and $U \in \tau$ if and only if $U \cap X_i \in \tau_i$ for each $i = 1, 2$. The free union of X_1 and X_2 is denoted by $X_1 + X_2$. If A is a non-empty closed subset of X_1, $f : A \to X_2$ is a mapping and D is the partition of $X_1 + X_2$ given by $D = \{\{p\} \cup f^{-1}(p) : p \in f(A)\} \cup \{\{x\} : x \in X_1 + X_2 \setminus (A \cup f(A))\}$, the decomposition space thus obtained is denoted by $X_1 \cup_f X_2$ and it is called the *attached space*. If X and Y are disjoint continua, then the attached space $X \cup_f Y$ is a continuum ([15, Theorem 3.20]).

3. Pseudo-homotopy

In this section we will develop general facts concerning pseudo-homotopies.

Definition 1. Let X and Y be topological spaces and let $f,\ g : X \to Y$ be mappings. We say that f is *homotopic* to g (or f and g are homotopic, written by $f \simeq g$), if there exists a mapping $H : X \times I \to Y$ (where I is the unit interval), called *homotopy*, fulfilling $H(x, 0) = f(x)$ and $H(x, 1) = g(x)$ for each $x \in X$.

Definition 2. Let X and Y be topological spaces and let $f,\ g : X \to Y$ be mappings. We say that f is *pseudo-homotopic* to g (or f and g are pseudo-homotopic) if there exist a continuum C, points $a,\ b \in C$ and a mapping $H : X \times C \to Y$ fulfilling $H(x, a) = f(x)$ and $H(x, b) = g(x)$ for each $x \in X$. The continuum C is called *factor space*. The mapping H is called a *pseudo-homotopy* between f and g. We write $f \simeq_C g$ to say that f is pseudo-homotopic to g, where C denotes a factor space.

It is easy to verify that if $f \simeq_C g$ and there exist a continuum K, and an onto mapping from K to C, then $f \simeq_K g$. Moreover, if there are a continuum K' and an onto mapping from K' to some subcontinuum $C' \subset C$ such that $a,\ b \in C'$, then $f \simeq_{K'} g$. Recall that two continua X and Y are said to be *continuously equivalent* provided that there are two onto mappings $f : X \to Y$ and $g : Y \to X$. So if C and D are continuously

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