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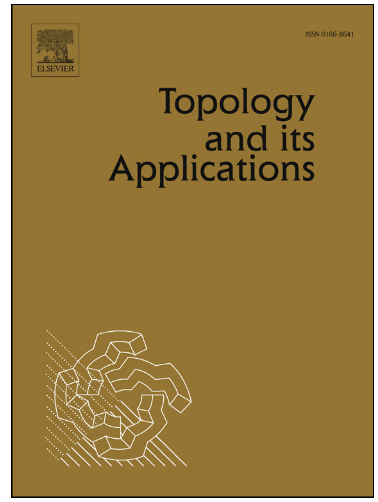

Group action on local dendrites

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GROUP ACTION ON LOCAL DENDRITES

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ABSTRACT. Let G be a group acting by homeomorphisms on a local dendrite X with countable set of endpoints. In this paper, it is shown that any minimal set M of G is either a finite orbit, or a Cantor set or a circle. Furthermore, we prove that if G is a finitely generated group, then the flow (G, X) is a pointwise recurrent flow if and only if one of the following two statements holds:

(1) $X = \mathbb{S}^1$, and $(G, \mathbb{S}^1)$ is a minimal flow conjugate to an isometric flow, or to a finite cover of a proximal flow;
(2) (G, X) is pointwise periodic.

1. INTRODUCTION

One of the oldest and interesting problem, in topological dynamics, is to find relations between (1) pointwise recurrence, (2) periodicity, (3) almost periodicity, (4) the orbit closure relation is closed, and (5) equicontinuity in the setting of a group G acting by homeomorphisms on a compact metric space (X, d). The pair (G, X) is called a flow. For $x \in X$, $Gx = \{gx : g \in G\}$ is called the orbit of x. Recall that, the orbit closure relation R is defined by $(x, y) \in R$ if $y \in \overline{Gx}$. Note that the relation R is an equivalence relation if and only if (G, X) is a pointwise almost periodic flow. The equicontinuous flows are dynamically the simplest ones; indeed, there is a complete classification of equicontinuous minimal flows [3]. It follows immediately from the definition that (2) implies (3) and (1). By [4, Proposition 1.1], (3) implies (1) and by [4, Proposition 1.7], (4) implies (1). In general, the equivalence of (4) and (5) as mentioned in [3, Exercise 6. page 46]), does not hold. In fact, as suggested by the referee, there is a flow on a compact metrizable space which is not equicontinuous such that the orbit closure relation is closed. Indeed, let (y_n) be a sequence converging to $y_\infty \in \mathbb{R} \setminus \mathbb{Q}$ with $y_n \in \mathbb{R} \setminus \mathbb{Q}$, $\mathbb{S}^1 = \mathbb{R}/\mathbb{Z}$ be a unit circle, and $X = \mathbb{S}^1 \times \{y_n : n \in \mathbb{N} \cup \{\infty\}\}$. Then X is a compact metrizable space with the metric d induced from one of the Euclidean space $\mathbb{R}^2$. Define the homeomorphism $f : X \to X$ by $f(x, y) = (x + y, y)$. Let G be a $\mathbb{Z}$-action inducted by f. Then G is not equicontinuous but the closure relation R is

$$\bigcup_{(x,y)\in X} \{(x, y)\} \times (\mathbb{S}^1 \times \{y\})$$

Indeed, fix a small $\varepsilon > 0$ (e.g. $\varepsilon = \frac{1}{4}$) and arbitrary $\delta > 0$. Choose a pair $p = (x, y)$; $q = (x, y_\infty)$ with $0 < |y - y_\infty| < \varepsilon$ and $d(q, p) < \delta$. Fix an integer $N > 1$ such that $k|y - y_\infty| < \varepsilon < N|y - y_\infty| < 1$ for any integer $k < N$. Then $d(f^N(p), f^N(q)) > N|y - y_\infty| > \varepsilon$. Consequently, G is not equicontinuous.

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