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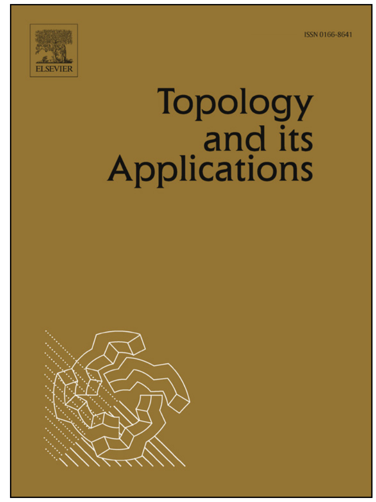

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$PSL_2(\mathbb{C})$-character varieties and Seifert fibered cosmetic surgeries

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Abstract

We study small Seifert possibly chiral cosmetic surgeries on not necessarily null-homologous knot in rational homology spheres. We define an absolute semi-norm derived from the Culler-Shalen semi-norm of $PSL_2(\mathbb{C})$-character variety theory. We prove that two cosmetic slopes must have the same absolute semi-norm and use this result to give a sharp bound on the number of slopes producing the same small Seifert manifold if the ambient manifold satisfies some representation theoretic conditions.

Keywords: Dehn surgeries, cosmetic surgeries, character variety.
2000 MSC: 57M99, 57M25, 57M27, 57M50

1. Introduction

Let Y be a rational homology sphere and K be a knot in Y such that $Y_K := Y \setminus int(\mathcal{N}(K))$ is *boundary irreducible* and *irreducible*. Two Dehn surgeries $Y_K(r)$ and $Y_K(s)$ with distinct slopes are called *cosmetic* if they are homeomorphic. They are called *truly cosmetic* if the homeomorphism preserve orientation and *chirally cosmetic* if the homeomorphism reverse orientation. The main conjecture about cosmetic surgery is the following [11].

Conjecture 1.1 (**Cosmetic surgery conjecture).** If two Dehn surgeries $Y_K(r)$ and $Y_K(s)$ with distinct slopes r and s are homeomorphic, then the homeomorphism reverses orientation.

In this paper we will focus on *general* cosmetic surgery and will not distinguish between chiral and truly cosmetic surgeries.

Let's fix a slope s and define

$$C_K(s) = \{\text{slope } \ r \neq s | \ Y_K(r) \cong Y_K(s)\} .$$

Here we allow the homeomorphism to reverse the orientation. If $C_K(s) \neq \emptyset$ then we have a cosmetic surgery (possibly chiral). The following theorem then gives a bound on the number of element in $C_K(s)$.

Theorem 1.2. Let K be a small knot in Y and $Y_K(s)$ be small-Seifert. If $\text{Hom}\,(\pi_1(Y), PSL_2(\mathbb{C}))$ contains only diagonalisable representations and $||s||$ is not a multiple of $s \cdot \lambda$ then $\sharp C_K(s) \leq 1$.

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