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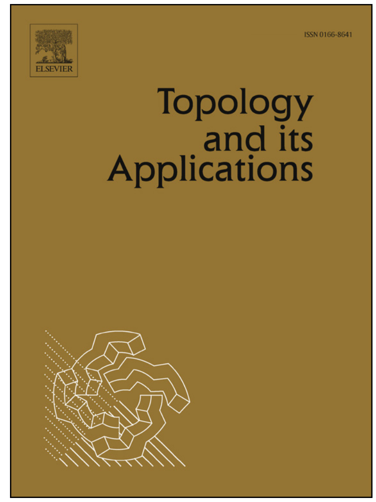

topological entropy on closed sets in $[0, 1]^2$

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TOPOLOGICAL ENTROPY ON CLOSED SETS IN $[0,1]^2$

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Abstract. We generalize the definition of topological entropy due to Adler, Konheim, and McAndrew [AKM] to set-valued functions from a closed subset A of the interval to closed subsets of the interval. We view these set-valued functions, via their graphs, as closed subsets of $[0,1]^2$. We show that many of the topological entropy properties of continuous functions of a compact topological space to itself hold in our new setting, but not all. We also compute the topological entropy of some examples, relate the entropy to other dynamical and topological properties of the examples, and we give an example of a closed subset G of $[0,1]^2$ that has 0 entropy but $G \cup \{(p,q)\}$, where $(p,q) \in [0,1]^2 \setminus G$, has infinite entropy.

1. Introduction

In this paper we generalize the idea of topological entropy to closed subsets of $[0,1]^2$, and later to closed subsets of $[0,1]^n$, for n a positive integer greater than 1. We reduce the problem of computing topological entropy in our context to one of counting the "boxes" (elements of our grid covers) certain sets generated by our closed subset of $[0,1]^n$ intersect. We also relate the topological entropy of the examples we give to the topology and dynamics of the examples.

The original definition of topological entropy was given by Adler, Konheim and McAndrew [AKM], and was defined for continuous maps on compact metric spaces. In [Bo2], R. E. Bowen showed that the topological entropy of the shift map σ on $\varprojlim f$ is equal to the topological entropy of f, where topological entropy of a continuous function on a compact metric space has that original definition given in [AKM]. Later came the contributions of Dinaburg [Di] and Bowen [Bo1], in which topological entropy was extended to a larger class of continuous maps, and defined differently. However, these two defintions of topological entropy for a continuous map on a compact metric space coincide. James Kelly and Tim Tennant [KT] have recently studied topological entropy of set-valued functions using Bowen's definition. Our focus is different, as we use the original definition using open covers due to Adler, Konheim and McAndrew [AKM]. There is some overlap of our results with those in [KT], and we point these out as they occur. Our results do agree with theirs, and by Theorem 4.3 of this paper and Theorem 3.1 of [KT], the two definitions are equivalent for closed subsets of $[0,1]^n$.

Suppose X is a compact metric space. Recall that if $f : X \to X$ is a continuous function, the *inverse limit space* generated by f is

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