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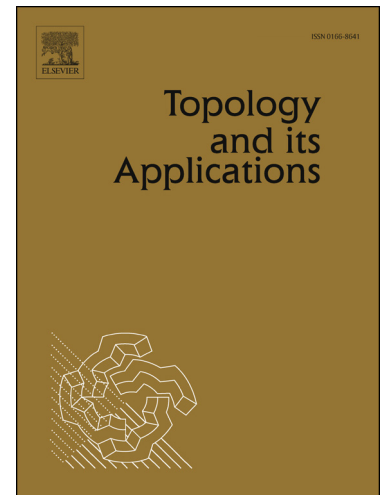
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# Shy Maps in Topology

Laurence Boxer \*

## Abstract

There is a concept in digital topology of a *shy map*. We define an analogous concept for topological spaces: We say a function is shy if it is continuous and the inverse image of every path-connected subset of its image is path-connected. Some basic properties of such maps are presented. For example, every shy map onto a semilocally simply connected space induces a surjection of fundamental groups (but a shy map onto a space that is not semilocally simply connected need not do so).

Key words and phrases: digital topology, fundamental group, wedge

## 1 Introduction

Shy maps between digital images were introduced in [2] and studied in subsequent papers belonging to the field of digital topology, including [3, 7, 4, 5, 6]. In this paper, we develop an analogous notion of a shy map between topological spaces and study its properties.

Recall that if  $F : X \rightarrow Y$  is a continuous function of topological spaces such that  $F(x_0) = y_0$ , then  $F$  induces a homomorphism of fundamental groups,  $F_* : \Pi_1(X, x_0) \rightarrow \Pi_1(Y, y_0)$ , defined by  $F_*([f]) = [F \circ f]$  for every loop  $f : (S^1, s_0) \rightarrow (X, x_0)$ , where  $S^1$  is the unit circle in the Euclidean plane.

A topological space  $X$  is *semilocally simply connected* [8] if for every  $x \in X$  there is a neighborhood  $N_x$  of  $x$  in  $X$  such that every loop in  $N_x$  is nullhomotopic in  $X$ .

We let  $\mathbb{Z}$  denote the set of integers, and  $\mathbb{R}$ , the real line.

A digital image is often considered as a graph  $(X, \kappa)$ , where  $X \subset \mathbb{Z}^n$  for some positive integer  $n$  and  $\kappa$  is an adjacency relation on  $X$ . A function  $f : (X, \kappa) \rightarrow (Y, \lambda)$  between digital images is *continuous* if for every  $\kappa$ -connected subset  $A$  of  $X$ ,  $f(A)$  is a  $\lambda$ -connected subset of  $Y$  [16, 1]. A continuous surjection  $f : (X, \kappa) \rightarrow (Y, \lambda)$  between digital images is called *shy* [2] if the following hold.

- For all  $y \in Y$ ,  $f^{-1}(y)$  is a  $\kappa$ -connected subset of  $X$ , and
- for all pairs of  $\lambda$ -adjacent  $y_0, y_1 \in Y$ ,  $f^{-1}(\{y_0, y_1\})$  is a  $\kappa$ -connected subset of  $X$ .

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