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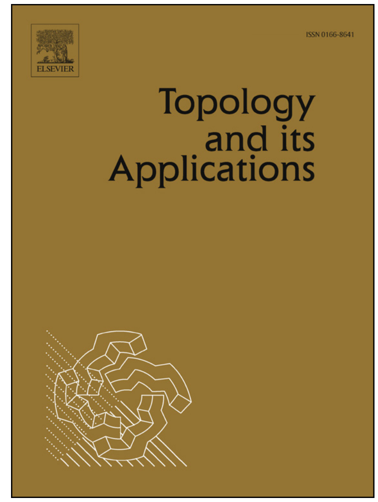

Shy Maps in Topology

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Shy Maps in Topology

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Abstract

There is a concept in digital topology of a *shy map*. We define an analogous concept for topological spaces: We say a function is shy if it is continuous and the inverse image of every path-connected subset of its image is path-connected. Some basic properties of such maps are presented. For example, every shy map onto a semilocally simply connected space induces a surjection of fundamental groups (but a shy map onto a space that is not semilocally simply connected need not do so).

Key words and phrases: digital topology, fundamental group, wedge

1 Introduction

Shy maps between digital images were introduced in [2] and studied in subsequent papers belonging to the field of digital topology, including [3, 7, 4, 5, 6]. In this paper, we develop an analogous notion of a shy map between topological spaces and study its properties.

Recall that if $F : X \to Y$ is a continuous function of topological spaces such that $F(x_0) = y_0$, then F induces a homomorphism of fundamental groups, $F_* : \Pi_1(X, x_0) \to \Pi_1(Y, y_0)$, defined by $F_*([f]) = [F \circ f]$ for every loop $f : (S^1, s_0) \to (X, x_0)$, where S^1 is the unit circle in the Euclidean plane.

A topological space X is *semilocally simply connected* [8] if for every $x \in X$ there is a neighborhood N_x of x in X such that every loop in N_x is nullhomotopic in X.

We let $\mathbb{Z}$ denote the set of integers, and $\mathbb{R}$, the real line.

A digital image is often considered as a graph (X, κ), where $X \subset \mathbb{Z}^n$ for some positive integer n and κ is an adjacency relation on X. A function $f : (X, \kappa) \to (Y, \lambda)$ between digital images is *continuous* if for every κ-connected subset A of X, $f(A)$ is a λ-connected subset of Y [16, 1]. A continuous surjection $f : (X, \kappa) \to (Y, \lambda)$ between digital images is called *shy* [2] if the following hold.

- For all $y \in Y$, $f^{-1}(y)$ is a κ-connected subset of X, and
- for all pairs of λ-adjacent $y_0, y_1 \in Y$, $f^{-1}(\{y_0, y_1\})$ is a κ-connected subset of X.

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