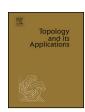


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Seifert surgery on knots via Reidemeister torsion and Casson–Walker–Lescop invariant III



Teruhisa Kadokami ^{a,*}, Noriko Maruyama ^b, Tsuyoshi Sakai ^c

- ^a School of Mechanical Engineering, College of Science and Engineering, Kanazawa University, Kakuma-machi, Kanazawa, Ishikawa, 920-1192, Japan
- b Musashino Art University, Ogawa 1-736, Kodaira, Tokyo 187-8505, Japan
- ^c Department of Mathematics, Nihon University, 3-25-40, Sakurajosui, Ŝetagaya-ku, Tokyo 156-8550, Japan

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ABSTRACT

For a knot K in a homology 3-sphere Σ , let M be the result of 2/q-surgery on K, and let X be the universal abelian covering of M. Our first theorem is that if the first homology of X is finite cyclic and M is a Seifert fibered space with $N \geq 3$ singular fibers, then $N \geq 4$ if and only if the first homology of the universal abelian covering of X is infinite. Our second theorem is that under an appropriate assumption on the Alexander polynomial of K, if M is a Seifert fibered space, then $q = \pm 1$ (i.e. integral surgery).

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1. Introduction

It is conjectured that Seifert surgeries on non-trivial knots are integral (except some cases), and that the resultant Seifert fibered spaces have three singular fibers (for example, see [2]). We [5,6] have studied the 2/q-Seifert surgeries. One reason why the coefficients are 2/q is that we started from the following shown in [3]:

Theorem 1.1. ([3, Theorem 1.4]) Let K be a knot in a homology 3-sphere Σ such that the Alexander polynomial of K is $t^2 - 3t + 1$. The only surgeries on K that may produce a Seifert fibered space with

E-mail addresses: kadokami@se.kanazawa-u.ac.jp (T. Kadokami), maruyama@musabi.ac.jp (N. Maruyama).

^{*} Corresponding author.

base S^2 and with $H_1 \neq \{0\}$, \mathbb{Z} have coefficients 2/q and 3/q, and produce Seifert fibered space with three singular fibers. Moreover (1) if the coefficient is 2/q, then the set of multiplicities is $\{2\alpha, 2\beta, 5\}$ where $\gcd(\alpha, \beta) = \gcd(\alpha, 5) = \gcd(\beta, 5) = 1$, and (2) if the coefficient is 3/q, then the set of multiplicities is $\{3\alpha, 3\beta, 4\}$ where $\gcd(\alpha, \beta) = \gcd(\alpha, 2) = \gcd(\beta, 2) = 1$.

In this paper, we continue to study the 2/q-Seifert surgeries, where we replace the condition on the Alexander polynomial with a condition on the first homology of the double branched covering. We note that the results below are related to the conjectures mentioned above.

- (1) Let Σ be a homology 3-sphere, and let K be a knot in Σ . Then $\Delta_K(t)$ denotes the Alexander polynomial of K, $a_2(K)$ denotes the Conway's a_2 -term of K, and $\Sigma(K; p/r)$ denotes the result of p/r-surgery on K.
- (2) The first author [4] introduced the norm of polynomials and homology lens spaces: Let ζ_d be a primitive d-th root of unity. For an element α of $\mathbb{Q}(\zeta_d)$, $N_d(\alpha)$ denotes the norm of α associated to the algebraic extension $\mathbb{Q}(\zeta_d)$ over \mathbb{Q} . Let f(t) be a Laurent polynomial over \mathbb{Z} . We define $|f(t)|_d$ by

$$|f(t)|_d = |N_d(f(\zeta_d))| = \left| \prod_{i \in (\mathbb{Z}/d\mathbb{Z})^{\times}} f(\zeta_d^i) \right|.$$

Let X be a homology lens space with $H_1(X) \cong \mathbb{Z}/p\mathbb{Z}$. Then there exists a knot K in a homology 3-sphere Σ such that $X = \Sigma(K; p/r)$ ([1, Lemma 2.1]). We define $|X|_d$ and $||X||_d$ by

$$|X|_d = |\Delta_K(t)|_d$$
 and $||X||_d = \prod_{d'|d,d' \ge 1} |X|_{d'}$,

where d is a divisor of p. Then both $|X|_d$ and $||X||_d$ are topological invariants of X (refer to [4] for details). (3) Let X be a closed oriented 3-manifold. Then $\lambda(X)$ denotes the Lescop invariant of X ([7]). Note that $\lambda(S^3) = 0$.

2. Results

Let K be a knot in a homology 3-sphere Σ . For an odd integer q, let M be the result of 2/q-surgery on K: $M = \Sigma(K; 2/q)$. Let $\pi: X \to M$ be the universal abelian covering of M (i.e. the covering associated to $\operatorname{Ker}(\pi_1(M) \to H_1(M))$). Since $H_1(M) \cong \mathbb{Z}/2\mathbb{Z}$, π is the 2-fold unbranched covering.

Let Σ_2 be the double branched covering space of Σ branched along K, and \overline{K} the lifted knot of K in Σ_2 . Since \overline{K} is null-homologous in Σ_2 , and X is the result of 1/q-surgery on \overline{K} , we have $H_1(X) \cong H_1(\Sigma_2)$.

$$\overline{K} \subset \Sigma_2 \sim X = \Sigma_2(\overline{K}; 1/q)$$

$$\downarrow \qquad \qquad \downarrow$$

$$K \subset \Sigma \sim M = \Sigma(K; 2/q)$$

Assume that $H_1(\Sigma_2) \cong \mathbb{Z}/m\mathbb{Z}$ with $m \geq 3$. Then $H_1(X) \cong \mathbb{Z}/m\mathbb{Z}$. Hence $|X|_d$ is defined for each divisor d of m, and so is $||X||_d$.

We then have the following.

Theorem 2.1. Let K be a knot in a homology 3-sphere Σ . With the notation above, we assume that $H_1(\Sigma_2) \cong \mathbb{Z}/m\mathbb{Z}$ with $m \geq 3$, and $M = \Sigma(K; 2/q)$ is a Seifert fibered space with N singular fibers where $N \geq 3$. Then the following hold.

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