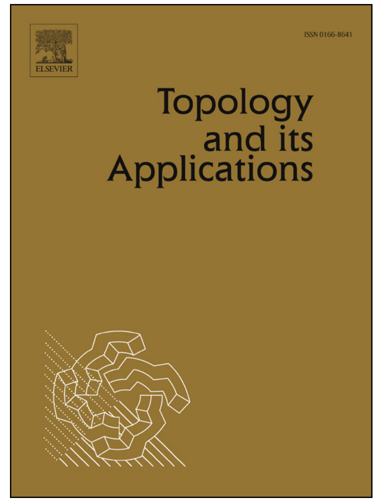

Warsaw discs and semicomputability

Zvonko Iljazović, Bojan Pažek

PII: S0166-8641(18)30104-4
DOI: https://doi.org/10.1016/j.topol.2018.02.014
Reference: TOPOL 6399

To appear in: *Topology and its Applications*

Received date: 5 February 2017
Revised date: 21 July 2017

Please cite this article in press as: Z. Iljazović, B. Pažek, Warsaw discs and semicomputability, *Topol. Appl.* (2018), https://doi.org/10.1016/j.topol.2018.02.014

Warsaw discs and semicomputability

Zvonko Iljazović

Department of Mathematics, Faculty of Science, University of Zagreb, Croatia

Bojan Pažek

Faculty of Architecture, University of Zagreb, Croatia

Abstract

We examine conditions under which a semicomputable set in a computable metric space is computable. Topology plays an important role in the description of such conditions. Motivated by the known result that a semicomputable cell is computable if its boundary sphere is computable, we investigate semicomputable Warsaw discs and their boundary Warsaw circles. We prove that a semicomputable Warsaw disc is computable if its boundary Warsaw circle is semicomputable.

Keywords: computable metric space, computable set, semicomputable set, Warsaw circle, Warsaw disc
2010 MSC: 03F60, 54H99

1. Introduction

A point $x \in \mathbb{R}^n$ is computable if it can be effectively approximated by a rational point $q \in \mathbb{Q}^n$ with arbitrary precision. A nonempty compact subset S of $\mathbb{R}^n$ is computable if it can be effectively approximated by a finite subset A of $\mathbb{Q}^n$, in the sense of the Hausdorff metric, with arbitrary precision. Each computable set contains computable points, moreover they are dense in it.

On the other hand, a nonempty compact subset S of $\mathbb{R}^n$ is called semicomputable if we can effectively enumerate all rational open sets (i.e. finite unions of open balls whose centers are rational points and radii are rational numbers) which cover S. This is equivalent to saying that $S = f^{-1}(\{0\})$ for some computable function $f : \mathbb{R}^n \to \mathbb{R}$. Each computable set is semicomputable, but a semicomputable set need not be computable. In fact, there exists a computable function $f : \mathbb{R} \to \mathbb{R}$ which has zero-points and they are all contained in $[0, 1]$, but none of them is computable, meaning that $f^{-1}(\{0\})$ is a semicomputable set which contains no computable point, hence which is "far away from being computable".

Nevertheless, under certain conditions a semicomputable set is computable. Some topological properties can force a semicomputable set to be computable