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A construction of slice knots via annulus modifications

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ABSTRACT

We define an operation on a homology B^4 that we call an n-twist annulus modification. We give a new construction of smoothly slice knots and exotically slice knots via n-twist annulus modifications. As an application, we present a new example of a smoothly slice knot with non-slice derivatives. Such examples were first discovered by Cochran and Davis. Also, we relate n-twist annulus modifications to n-fold annulus twists which was first introduced by Osoinach and has been used by Abe and Tange to construct smoothly slice knots. Furthermore we show that any exotic slice disk can be obtained by an annulus modification performed on some exotic slice disk bounding the unknot.

1. Introduction

We begin with some definitions. A knot K is an embedding of an oriented S^1 into S^3 and a link L is an embedding of a disjoint collection of oriented S^1 into S^3. If K bounds a smoothly embedded 2-disk D^2 in the standard B^4, we call K a smoothly slice knot and D^2 a smooth slice disk. Moreover, if K bounds a smoothly embedded 2-disk D^2 in a smooth oriented 4-manifold M that is homeomorphic to the standard B^4 but not necessarily diffeomorphic to the standard B^4, we call K exotically slice in M and D^2 a exotic slice disk. Note that we can define a radial function on B^4 and by a small isotopy of D^2 one can ensure that the radial function restricts to a Morse function on D^2. If a knot K bounds a smoothly embedded 2-disk D^2 in the standard B^4 where there are no local maxima of the radial function restricted to D^2, we call K a ribbon knot. A link L is a smoothly slice link if each component of L bounds a smoothly embedded disjoint 2-disk D^2 in the standard B^4.

In this paper we will define an operation on homology 4-balls that we call an n-twist annulus modification in Section 2. Then we will fix a slice knot and use this modification under a certain condition (see Section 2), to obtain an infinite family of exotically slice knots. The basic idea for this condition, which is called ℓ-nice,

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$B^4 = B^3 \times [0,1] =$

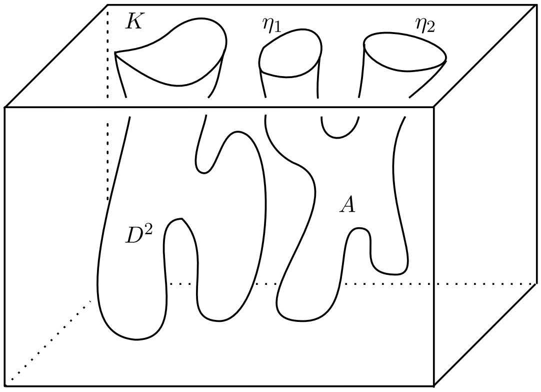

Fig. 1. Schematic picture of the assumptions of Theorem 2.3.

is that there exist a smooth proper embedding of an annulus in B^4 such that the result of Dehn surgery along the interval is homeomorphic to B^4. In addition, if there is a smooth slice disk disjoint from the annulus, then the image of the knot under the modification is exotically slice (see Fig. 1). The technique we use here is similar to the technique that was used in [3] to construct a smoothly slice knot with non-slice derivatives (see Section 1.1).

Theorem 2.3. *Let K be a smoothly slice knot, ϕ_A be ℓ-nice, and $\eta_1 \cup -\eta_2$ be the boundary of A. Suppose there exists a smoothly embedded slice disk for K in the complement of A. Then $\frac{n\ell+1}{n}$ Dehn surgery on η_1 followed by $\frac{n\ell-1}{n}$ Dehn surgery on η_2 produces an exotically slice knot $K_{(\phi_A,n)} \subset S^3$.*

If we restrict our condition further (see Section 3), which is called ℓ-standard, we can use annulus modifications to obtain a infinite family of smoothly slice knots. The condition is that the smooth proper embedding of an annulus is isotopic to the standard annulus, denoted A_ℓ and shown in Fig. 7.

Theorem 3.3. *Let K be a smoothly slice knot, ϕ_A be ℓ-standard, and $\eta_1 \cup -\eta_2$ be the boundary of A. Suppose there exists a smoothly embedded slice disk for K in the complement of A. Then $\frac{n\ell+1}{n}$ Dehn surgery on η_1 followed by $\frac{n\ell-1}{n}$ Dehn surgery on η_2 produces a smoothly slice knot $K_{(\phi_A,n)} \subset S^3$.*

We have two applications of these theorems.

1.1. Application 1: an example of a slice knot with non-slice derivatives

Recall that any knot in S^3 bounds a Seifert Surface F. From F, we can define a Seifert form $\beta_F : H_1(F) \times H_1(F) \to \mathbb{Z}$, which is defined by $\beta_F([x],[y]) = \mathrm{lk}(x, y^+)$, where x is a union of simple closed curves on F that represents $[x]$, y^+ is a positive push off of union of simple closed curves on F that represents $[y]$, and lk denotes linking number. It was proven by Levine [16, Lemma 2] that if K is a smoothly slice knot then β_F is metabolic for any Seifert surface F for K, i.e. there exists $H = \mathbb{Z}^{\frac{1}{2}\operatorname{rank} H_1(F)}$, a direct summand of $H_1(F)$, such that β_F vanishes on H. We call a knot algebraically slice if it has metabolic Seifert form. Then a link $\{\gamma_1 \cup \gamma_2 \cup \cdots \cup \gamma_{\frac{1}{2}\operatorname{rank} H_1(F)}\}$ disjointly embedded in a surface F where its homology classes form a basis for H is called a derivative of K (see Fig. 2). Notice that we can define a derivative of a knot for any algebraically slice knot.

If a knot K has a derivative which is a smoothly slice link, then K is smoothly slice (see Fig. 3). A natural question is whether the converse holds. This was asked by Kauffman in 1982, for genus 1 knots.

Conjecture 1.1. *[14,15, N1.52] If K is a smoothly slice knot and F is a genus 1 Seifert surface for K then there exists an essential simple closed curve d on F such that $\mathrm{lk}(d, d^+) = 0$ and d is a slice knot.*