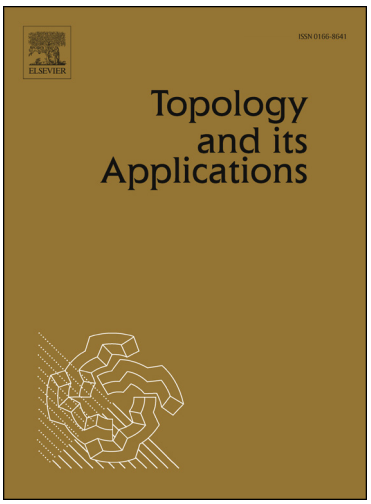
Accepted Manuscript



On the area of constrained polygonal linkages

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PII: S0166-8641(18)30089-0
DOI: https://doi.org/10.1016/j.topol.2018.02.004
Reference: TOPOL 6387

To appear in: *Topology and its Applications*

Received date: 24 February 2017
Revised date: 5 October 2017
Accepted date: 5 February 2018

Please cite this article in press as: G. Panina, D. Siersma, On the area of constrained polygonal linkages, *Topol. Appl.* (2018), https://doi.org/10.1016/j.topol.2018.02.004

ON THE AREA OF CONSTRAINED POLYGONAL LINKAGES

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Abstract. Consider a mechanical linkages whose underlying graph is a polygon with a diagonal constraint, or more general, a partial two-tree. We show that (with an appropriate definition) the oriented area is a Bott-Morse function on the configuration space of the linkage. Its critical points are described and Bott-Morse indices are computed.

This paper is a generalization of analogous results for polygonal linkages (obtained earlier by G. Khimshiashvili, G. Panina, and A. Zhukova).

1. Introduction

A *polygonal linkage* is a linkage whose underlying graph is a polygon (or, equivalently, a single-cycle graph). One thinks of it as of a flexible polygon with rigid edges and revolving joints at the vertices whose ambient space is the Euclidean plane. The idea of considering the oriented area as a Morse function on its configuration space has already led to some non-trivial results: the critical points (or, equivalently, critical configurations) are easily describable, and there exists a short formula for the Morse index [6], [8], [9], [13]. In some further generalization [10] the oriented area proves to be an exact Morse function.

In the present paper we extend the class of underlying graphs of linkages in such a way that it is possible to introduce the oriented area with the same nice Morse-theoretical properties.

We start in Section 3 with *three-chain linkages.* It is our first example which is not a polygonal linkage. By definition, a three-chain linkage is a patch of three chains, and therefore the underlying graph has three cycles. One of the cycles is distinguished: we consider its area S as the function defined on the configuration space. Generically, it is a Bott-Morse function. We prove that the critical configurations are characterized by a combination of *cyclic* and *aligned* conditions. We also give a formula for Bott-Morse indices of critical points and critical components (Section 4).

As an interesting illustration, we describe a Hessian bifurcation and show that it amounts to a pitchfork bifurcation (Section 5).

2000 *Mathematics Subject Classification.* 52R70, 52B99.

Key words and phrases. Morse index, critical point, partial two-tree, two-terminal series-parallel graph, pitchfork bifurcation.