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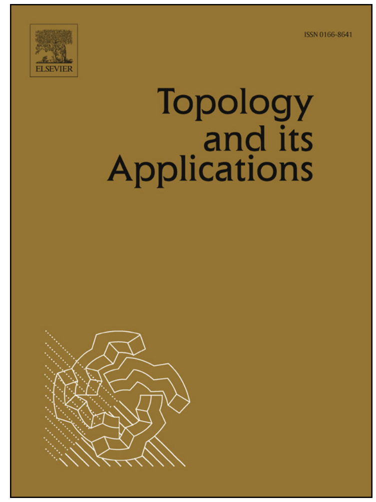

Real-valued functions and some related spaces

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REAL-VALUED FUNCTIONS AND SOME RELATED SPACES

ER-GUANG YANG

ABSTRACT. We list some conditions imposed on real-valued functions and show that a large number of spaces, such as first countable spaces, γ-spaces, Nagata-spaces, semi-metrizable spaces, quasi-metrizable spaces as well as metrizable spaces can be characterized with real-valued functions satisfying one or more of these conditions.

1. Introduction and preliminaries

Throughout, a space always means a topological space. For a space X, we denote by $\mathcal{C}_X$ the family of all compact subsets of X. τ and τ^c denote the topology of X and the family of all closed subsets of X, respectively. For a subset A of a space X, we write $\overline{A}$ (resp., $intA$) for the closure (resp., interior) of A in X. Also, we use χ_A to denote the characteristic function of A. An open set U is called an open neighborhood of a set A if $A \subset U$. The set of all positive integers is denoted by $\mathbb{N}$ while $\langle x_n \rangle$ denotes a sequence.

A real-valued function f on a space X is called *lower* (resp., *upper*) *semi-continuous* [7] if for any real number r, the set $\{x \in X : f(x) > r\}$ (resp., $\{x \in X : f(x) < r\}$) is open. f is called *k-lower semi-continuous* [28] if for each $K \in \mathcal{C}_X$, f has a minimum value on K. It is known that [28] a lower semi-continuous function is k-lower semi-continuous. We write $L(X)$ (resp., $U(X)$, $KL(X)$) for the set of all lower (resp., upper, k-lower) semi-continuous functions from X into the unit interval $[0,1]$. $UKL(X) = U(X) \cap KL(X)$. $C(X)$ is the set of all continuous functions from X into $[0,1]$.

Definition 1.1. A space X is called *stratifiable* [3] (*semi-stratifiable* [5]) if there is a map $\rho : \mathbb{N} \times \tau^c \to \tau$ such that
(1) $F = \bigcap_{n\in\mathbb{N}} \rho(n,F) = \bigcap_{n\in\mathbb{N}} \overline{\rho(n,F)}$ ($F = \bigcap_{n\in\mathbb{N}} \rho(n,F)$) for each $F \in \tau^c$;
(2) if $F, H \in \tau^c$ and $F \subset H$, then $\rho(n,F) \subset \rho(n,H)$ for all $n \in \mathbb{N}$.

Let $A, B \subset X$ and f_A a real-valued function on X related to A. For a singleton $\{x\}$, we write f_x instead of $f_{\{x\}}$. Consider the following conditions:
(e_A) $A = f_A^{-1}(0)$.
(m_A) If $A_1 \subset A_2$, then $f_{A_1} \geq f_{A_2}$.
(i_{AB}) If $A \cap B = \emptyset$, then $\inf\{f_A(x) : x \in B\} > 0$.
(i'_{AB}) If $A \cap B = \emptyset$, then there exists an open neighborhood V of B such that $\inf\{f_A(x) : x \in V\} > 0$.

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