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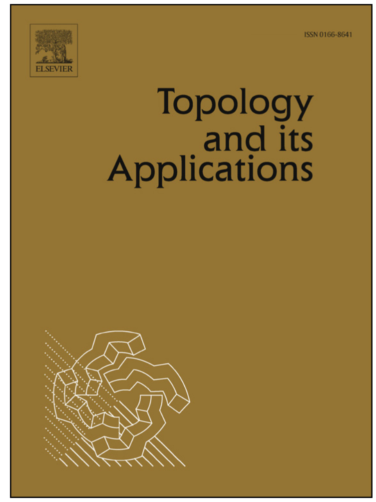

Classification of metric spaces with infinite asymptotic dimension

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Classification of metric spaces with infinite asymptotic dimension

Yan Wu* Jingming Zhu** *

Abstract. We introduce a geometric property called complementary-finite asymptotic dimension (coasdim). Similar to the case of asymptotic dimension, we prove the corresponding coarse invariant theorem, union theorem and Hurewicz-type theorem. Moreover, we show that $\mathrm{coasdim}(X) \leq \omega + k$ implies $\mathrm{trasdim}(X) \leq \omega + k$ and transfinite asymptotic dimension of the shift union $\mathrm{sh}\bigcup \bigoplus_{i=1}^{\infty} 2^i\mathbb{Z}$ is no more than $\omega + 1$. i.e. $\mathrm{trasdim}(\mathrm{sh}\bigcup \bigoplus_{i=1}^{\infty} 2^i\mathbb{Z}) \leq \omega + 1$.

Keywords Asymptotic dimension, Transfinite asymptotic dimension, Asymptotic property C;

[1]

1 Introduction

In coarse geometry, asymptotic dimension of a metric space is an important concept which was defined by Gromov for studying asymptotic invariants of discrete groups [7]. This dimension can be considered as an asymptotic analogue of the Lebesgue covering dimension. As a large scale analogue of W.E. Haver's property C in dimension theory, A. Dranishnikov introduced the notion of asymptotic property C in [5]. It is well known that every metric space with finite asymptotic dimension has asymptotic property C [4]. But the inverse is not true, which means that there exists some metric space X with infinite asymptotic dimension and asymptotic property C. Therefore how to classify the metric spaces with infinite asymptotic dimension into smaller categories becomes an interesting problem. T. Radul defined the trasfinite asymptotic dimension (trasdim) which can be viewed as a transfinite extension for asymptotic dimension and gave examples of metric spaces with trasdim$= \omega$ and with trasdim$= \infty$ (see [8]). He also proved $\mathrm{trasdim}(X) < \infty$ if and only if X has asymptotic property C for every metric space X. But whether there is a metric space X with $\omega <\mathrm{trasdim}(X) < \infty$ is still unknown so far.

In this paper, we introduce another approach to classify the metric spaces with infinite asymptotic dimension, which is called complementary-finite asymptotic dimension (coasdim), and give some examples of metric spaces with different complementary-finite asymptotic dimensions. Moreover, we prove some properties of complementary-finite asymptotic dimension and show that $\mathrm{coasdim}(X) \leq \omega + k$ implies $\mathrm{trasdim}(X) \leq \omega + k$ for every metric space X.

The paper is organized as follows: In Section 2, we recall some definitions and properties of transfinite asymptotic dimension. In Section 3, we introduce complementary-finite asymptotic dimension and give some examples of metric spaces with different complementary-finite asymptotic dimensions. Besides, we prove some properties of complementary-finite asymptotic dimension like coarse invariant theorem, union theorem and Hurewicz-type theorem. In Section 4, we investigate the relationship between complementary-finite asymptotic dimension and transfinite asymptotic dimension. We also give an example of a metric space I_ω with $\mathrm{coasdim}(I_\omega) = \omega$ and $\mathrm{coasdim}(I_\omega \times I_\omega) \leq \omega + n$ is not true for any $n \in \mathbb{N}$, while $\mathrm{trasdim}(I_\omega) = \mathrm{trasdim}(I_\omega \times I_\omega) = \omega$. Finally, in Section 5, we define the shift union of $\bigoplus_{i=1}^{\infty} 2^i\mathbb{Z}$ by $\mathrm{sh}\bigcup \bigoplus_{i=1}^{\infty} 2^i\mathbb{Z}$ and prove that transfinite asymptotic dimension of $\mathrm{sh}\bigcup \bigoplus_{i=1}^{\infty} 2^i\mathbb{Z}$ is no more than $\omega + 1$.

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