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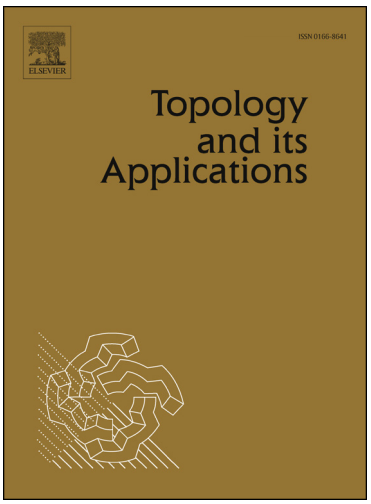

Pointfree pointwise convergence, Baire functions, and epimorphisms in truncated archimedean ℓ-groups

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POINTFREE POINTWISE CONVERGENCE, BAIRE FUNCTIONS, AND EPIMORPHISMS IN TRUNCATED ARCHIMEDEAN ℓ-GROUPS

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ABSTRACT. We define pointfree pointwise convergence, and use it to define the Baire functions on a locale. The main result is that the Baire functions on a locale coincide with the continuous functions on its P-locale coreflection. Furthermore, we show that the Baire functions on a locale constitute the epicompletion of the continuous functions in the relevant category.

The relevant category is **T**, the category of truncated archimedean ℓ-groups, hereafter nicknamed truncs. **T** is closely related to the famous category **W** of unital archimedean ℓ-groups. The universal objects in **T** are of the form $\mathcal{R}_0 L$, the trunc of real-valued locale maps $L \to \mathbb{R}$ which vanish at the designated point of a pointed locale L.

We provide an intuitive definition of pointwise convergence in $\mathcal{R}_0 L$ which extends the classical definition, and show that it has a number of nice properties: all homomorphisms and operations of **T** are pointwise continuous, and a pointwise dense extension is a trunc epimorphism. Conversely, we show that every epic extension $G \to H$ has an epic extension $H \to K$ such that G is pointwise dense in K.

We show that the rich theory of epimorphisms in **W** carries over to **T** with only minor modification. In particular, the epicomplete truncs comprise a full monoreflective subcategory, and are characterized as those objects of the form $\mathcal{R}_0 P$ for a P-locale P. In light of these facts, a reformulation of the last clause of the preceding paragraph is that any trunc is pointwise dense in any epicompletion. And a trunc is epicomplete iff it is pointwise complete, i.e., has no proper extension in which it is pointwise dense.

Finally, for a given pointed locale L, we define the functions of Baire class α on L in the classical fashion. A function is Baire class 0 if it lies in $\mathcal{R}_0 L$, and of Baire class β if it is the pointwise limit of a sequence of functions of Baire class $\alpha < \beta$. A Baire function on L is a function of Baire class α for some α. Our results can be summarized as follows.

Theorem. *For a pointed locale L with P-locale coreflection $\mathcal{P}_* L \to L$, the Baire functions on L are precisely the continuous functions on $\mathcal{P}_* L$, i.e., those of $\mathcal{R}_0 \mathcal{P}_* L$.*

Theorem. *The embedding $\mathcal{R}_0 L \to \mathcal{R}_0 \mathcal{P}_* L$ is the functorial epicompletion in* **T**.

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