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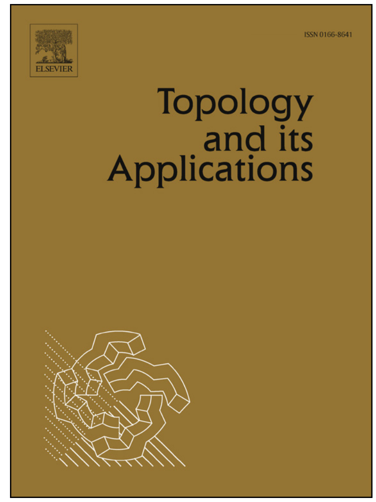

Regular Maps on Cartesian Products and Disjoint Unions of Manifolds

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Abstract

A map from a manifold to a Euclidean space is said to be k-regular if the images of any distinct k points are linearly independent. For k-regular maps on manifolds, lower bounds on the dimension of the ambient Euclidean space have been extensively studied. In this paper, we study the lower bounds on the dimension of the ambient Euclidean space for 2-regular maps on Cartesian products of manifolds. As corollaries, we obtain the exact lower bounds on the dimension of the ambient Euclidean space for 2-regular maps and 3-regular maps on spheres as well as on some real projective spaces. Moreover, generalizing the notion of k-regular maps, we study the lower bounds on the dimension of the ambient Euclidean space for maps with certain non-degeneracy conditions from disjoint unions of manifolds into Euclidean spaces.

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1 Introduction

Let M be a smooth manifold and let $\mathbb{F}$ denote the real numbers $\mathbb{R}$ or the complex numbers $\mathbb{C}$. For any $k \geq 2$, a map $f : M \longrightarrow \mathbb{F}^N$ is called (real or complex) k-regular if for any distinct k points $x_1, \ldots, x_k$ in M, $f(x_1), \ldots, f(x_k)$ are linearly independent in $\mathbb{F}^N$. For simplicity, a real k-regular map is also called a k-regular map. Any (real or complex) $(k+1)$-regular map is (real or complex) k-regular, and any k-regular map is injective.

Throughout this paper, all maps and functions are assumed to be continuous. We use S^m to denote the m-sphere and use $\mathbb{R}P^m$, $\mathbb{C}P^m$ and $\mathbb{H}P^m$ to denote the real, complex and quaternionic projective spaces consisting of real lines through the origin in $\mathbb{R}^{m+1}$, complex lines through the origin in $\mathbb{C}^{m+1}$ and quaternionic lines through the origin in $\mathbb{H}^{m+1}$ respectively. Moreover, we assume $m \geq 2$.

The study of k-regular maps was initiated in 1957 by K. Borsuk [4]. Later, the problem attracted additional attention because of its connection with the theory of Čebyšev approximation (cf. [11] and [22, pp. 237-242]):

> Theorem [Haar-Kolmogorov-Rubinstein]. Suppose M is compact and $f_1, \ldots, f_n$ are linearly independent real-valued functions on M. Let F be the linear space spanned by $f_1, \ldots, f_n$ over $\mathbb{R}$. Then $(f_1, \ldots, f_n)$ is a k-regular map from M to $\mathbb{R}^n$ if and only if for any real-valued function g on M, the dimension of the set $\{f \in F \mid \sup_{x\in M} |g(x) - f(x)| = m\}$ is smaller than or equal to $n-k$, where m is the infimum of $\sup_{x\in M} |g(x) - f(x)|$ for all f in F.

From 1970's to nowadays, k-regular maps on manifolds have been extensively studied. In 1978, some k-regular maps on the plane were constructed by F.R. Cohen and D. Handel [7]:

> [7, Example 1.2]. The map from $\mathbb{C}$ to $\mathbb{R}^{2k-1}$ sending z to $(1, z, z^2, \ldots, z^{k-1})$ is k-regular.

And in 1980, some 3-regular maps on spheres were constructed by D. Handel and J. Segal [15]:

> [15, Theorem 2.3]. Let i be the standard embedding from S^m to $\mathbb{R}^{m+1}$ and 1 the constant map with image 1. Then the map $(1, i)$ from S^m to $\mathbb{R}^{m+2}$ is 3-regular.

On the other hand, generalizing the results of M.E. Chisholm [6] in 1979 and F.R. Cohen and D. Handel [7] in 1978, the lower bounds of N for k-regular maps of Euclidean spaces into $\mathbb{R}^N$ were studied by P. Blagojević, W. Lück and G. Ziegler [1] in 2016. While the lower bounds of N for complex k-regular maps of Euclidean spaces into $\mathbb{C}^N$ were studied by P. Blagojević, F.R. Cohen, W. Lück and G. Ziegler [2] in 2015:

> [2, Theorem 5.2]. Let p be an odd prime. If there exists a complex p-regular map of $\mathbb{R}^m$ into $\mathbb{C}^N$, then $N \geq [\frac{m+1}{2}](p-1)+1$.
>
> [2, Theorem 5.3]. Let p be an odd prime and let $\alpha_p(k)$ be the sum of coefficients in the p-adic expansion of k. If m is a power of p and there exists a complex k-regular map of $\mathbb{C}^m$ into $\mathbb{C}^N$, then $N \geq m(k-\alpha_p(k)) + \alpha_p(k)$.