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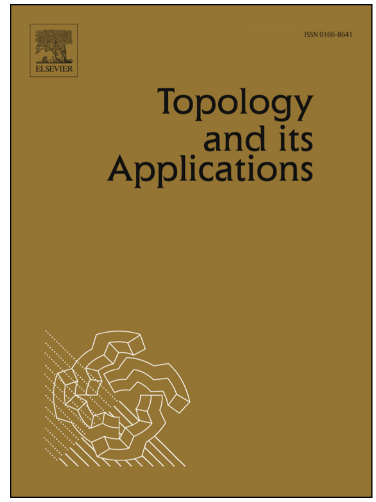

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Real intersection homology

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Abstract

We present a definition of intersection homology for real algebraic varieties that is analogous to Goresky and MacPherson's original definition of intersection homology for complex varieties.

Keywords: Real algebraic variety, intersection homology, arc-symmetric set, Nash approximation, stratification, small resolution, general position.
2010 MSC: 14P25, 14F43, 55N33, 57N80

Introduction

Let X be a real algebraic variety. For certain stratifications $\mathcal{S}$ of X we define homology groups $\mathrm{IH}_k^{\mathcal{S}}(X)$ with $\mathbb{Z}/2$ coefficients that generalize the standard intersection homology groups [3] if all strata have even codimension. Whether there is a good analog of intersection homology for real algebraic varieties was stated as a problem by Goresky and MacPherson [6] (Problem 7, p. 227). They observed that if such a theory exists then it cannot be purely topological; indeed our groups are not homeomorphism invariants.

We consider a class of algebraic stratifications introduced in [16] that have a natural general position property for semialgebraic subsets. After presenting the definition and properties of these stratifications $\mathcal{S}$, we define the real intersection homology groups $\mathrm{IH}_k^{\mathcal{S}}(X)$ and show that they are independent of the stratification. We prove that if X is nonsingular and pure dimensional then $\mathrm{IH}_k(X) = H_k(X;\mathbb{Z}/2)$, classical homology with $\mathbb{Z}/2$ coefficients. More generally, we prove that if X is irreducible and X admits a small resolution $\pi : \widetilde{X} \to X$ then $\mathrm{IH}_k(X)$ is canonically isomorphic to $H_k(\widetilde{X};\mathbb{Z}/2)$. Thus any two small resolution of X have the same homology.

If X is not compact, we have two versions of real intersection homology: $\mathrm{IH}_k^c(X)$ with compact supports and $\mathrm{IH}_k^{cl}(X)$ with closed supports. We define an intersection pairing $\mathrm{IH}_k^c(X) \times \mathrm{IH}_{n-k}^{cl}(X) \to \mathbb{Z}/2$, where $n = \dim X$. We prove that if X has isolated singularities this pairing is nonsingular, so $\mathrm{IH}_k^c(X) \cong \mathrm{IH}_{n-k}^{cl}(X)$ for all $k \geq 0$. But our intersection pairing is singular for some real algebraic varieties X,