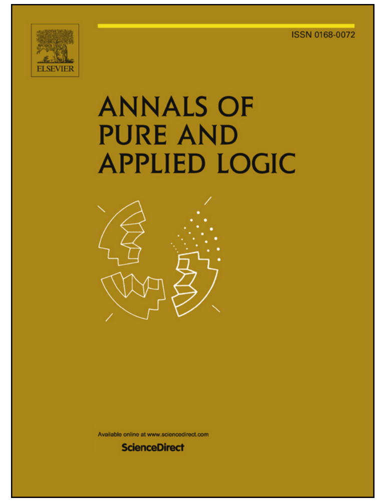

Topology and measure in logics for region-based theories of space

Tamar Lando

PII: S0168-0072(17)30143-4
DOI: https://doi.org/10.1016/j.apal.2017.12.002
Reference: APAL 2613

To appear in: *Annals of Pure and Applied Logic*

Received date: 24 May 2017
Revised date: 21 November 2017
Accepted date: 1 December 2017

Please cite this article in press as: T. Lando, Topology and measure in logics for region-based theories of space, *Ann. Pure Appl. Logic* (2018), https://doi.org/10.1016/j.apal.2017.12.002

Topology and Measure in Logics for Region-Based Theories of Space

Tamar Lando*

Abstract

Space, as we typically represent it in mathematics and physics, is composed of dimensionless, indivisible points. On an alternative, region-based approach to space, extended regions together with the relations of 'parthood' and 'contact' are taken as primitive; points are represented as mathematical abstractions from regions.

Region-based theories of space have been traditionally modeled in regular closed (or regular open) algebras, in work that goes back to de Laguna [1922] and Whitehead [1929]. More recently, formal logics for region-based theories of space were developed in, *e.g.*, Balbiani et al. [2007] and Vakarelov [2007]. It was shown that these logics have both a nice topological and relational semantics, and that the minimal logic for contact algebras, $\mathbb{L}^{cont}_{min}$ (defined below), is complete for both.

The present paper explores the question of completeness of $\mathbb{L}^{cont}_{min}$ and its extensions for individual topological spaces of interest: the real line, Cantor space, the rationals, and the infinite binary tree. A second aim is to study a different, algebraic model of logics for region-based theories of space, based on the Lebesgue measure algebra (or algebra of Borel subsets of the real line modulo sets of Lebesgue measure zero). As a model for point-free space, the algebra was first discussed in Arntzenius [2008]. The main results of the paper are that $\mathbb{L}^{cont}_{min}$ is weakly complete for any zero-dimensional, dense-in-itself metric space (including, *e.g.*, Cantor space and the rationals); the extension $\mathbb{L}^{cont}_{min} + (Con)$ is weakly complete for the real line and the Lebesgue measure contact algebra. We also prove that the logic $\mathbb{L}^{cont}_{min} + (Univ)$ is weakly complete for the infinite binary tree.

2010 Mathematics Subject Classification. 03B45; 03B20; 03B10
Key words and phrases. Region-based theories of space, contact algebras, modal logic, topological semantics, completeness, Lebesgue measure algebra

1 Introduction

For a long time, logicians have wondered whether our ways of representing space are in some sense too idealized. Euclidean space is made up of *points*: dimensionless, indivisible regions. These are the smallest parts of space—the atoms beyond which we can divide no further. But such spatial atoms do not seem to correspond to anything in our ordinary experience of the world. On an alternative, region-based approach to space, extended regions together with some mereological and topological relations are taken as primitive; points are constructed as mathematical abstractions from regions.

*Columbia University, Department of Philosophy, 708 Philosophy Hall, 1150 Amsterdam Avenue, Mail Code: 4971, New York, NY 10027, tal2108@columbia.edu