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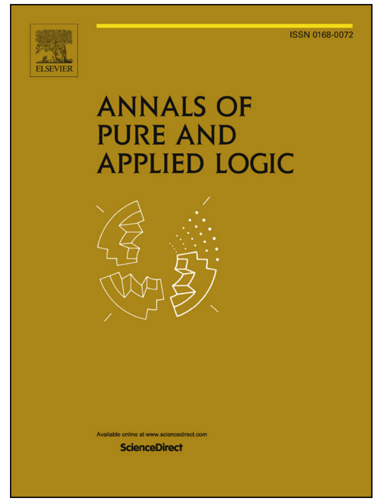

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WILD THEORIES WITH O-MINIMAL OPEN CORE

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Abstract. Let T be a consistent o-minimal theory extending the theory of densely ordered groups and let T' be a consistent theory. Then there is a complete theory T^* extending T such that T is an open core of T^*, but every model of T^* interprets a model of T'. If T' is NIP, T^* can be chosen to be NIP as well. From this we deduce the existence of an NIP expansion of the real field that has no distal expansion.

1. Introduction

Let $\mathcal{R}$ be an expansion of a dense linear order $(R, <)$ without endpoints. The **open core of** $\mathcal{R}$, denoted by $\mathcal{R}^\circ$, is the structure $(R, (U))$, where U ranges over all open sets of all arities definable in $\mathcal{R}$. Miller and Speissegger introduced this notion of an open core for expansions of $(\mathbb{R}, <)$ in [16], and established sufficient conditions on $\mathcal{R}$ such that its open core is o-minimal. Here we want to answer the following question:

Is there any restriction on what kind of structures can be interpreted in an expansion of $(R, <)$ with o-minimal open core?

This question, although formulated slightly differently, was already asked by Dolich, Miller and Steinhorn in a preprint version of [7]. Our answer is negative. To give a precise statement of our result, we need to recall the notion of an open core of a theory as introduced in [6]. Let T^* be a theory extending the theory of dense linear orders without endpoints in a language $\mathcal{L}^* \supseteq \{<\}$, and let T be another theory in a language $\mathcal{L}$. We say that T **is an open core of** T^* if for every $\mathcal{N} \models T^*$ there is $\mathcal{M} \models T$ such that $\mathcal{N}^\circ$ is interdefinable with $\mathcal{M}$.

Theorem A. Let T be a consistent o-minimal theory extending the theory of densely ordered groups and let T' be a consistent theory. Then there is a complete theory T^* extending T such that

(1) T^* interprets a model of T',
(2) T is an open core of T^*,
(3) T^* is NIP if T' is NIP,
(4) T^* is strongly dependent if T' is strongly dependent.

Statements (3) and (4) of Theorem A indicate that we can choose T^* in such a way that not only the open core of T^* is o-minimal, but also T^* remains tame in the sense of Shelah's combinatorial tameness notions. For definitions of NIP and

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