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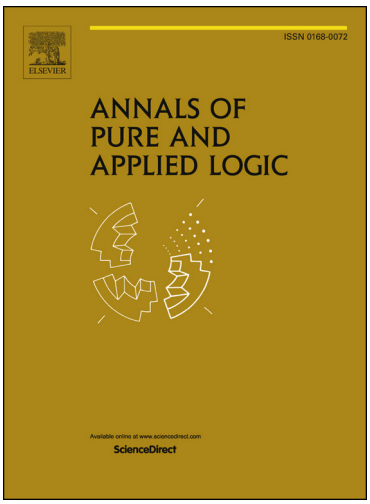

A computational glimpse at the Leibniz and Frege hierarchies

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A computational glimpse at the Leibniz and Frege hierarchies

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Abstract

In this paper we consider, from a computational point of view, the problem of classifying logics within the Leibniz and Frege hierarchies typical of abstract algebraic logic. The main result states that, for logics presented syntactically, this problem is in general undecidable. More precisely, we show that there is no algorithm that classifies the logic of a finite consistent Hilbert calculus in the Leibniz and in the Frege hierarchies.

Keywords: abstract algebraic logic, Leibniz hierarchy, Frege hierarchy Leibniz congruence, decidability, Diophantine equations, relation algebras
03G27, 03G15, 11Dxx

Dedicated to Professor Josep Maria Font on the occasion of his retirement

1. Introduction

Abstract algebraic logic (AAL for short) is a field that studies uniformly propositional logics [14, 11, 15, 16]. One of its main achievements is the development of the so-called Leibniz and Frege hierarchies in which propositional logics are classified according to two different criteria. More precisely, the *Leibniz hierarchy* provides a taxonomy that classifies propositional systems accordingly to the way their notions of *logical equivalence* and of *truth* can be defined. Roughly speaking, the location of a logic inside the Leibniz hierarchy reflects the strength of the relation that it enjoys with its algebraic counterpart. In this sense, the Leibniz hierarchy revealed to be a useful

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