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# Direct twisted Galois stratification

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ABSTRACT

The theory ACFA admits a primitive recursive quantifier elimination procedure. It is therefore primitive recursively decidable.

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## 1. Introduction

### 1.1. Background

The role of our work in Model Theory of fields with powers of Frobenius and existentially closed difference fields is analogous to the role that Galois stratification of Fried, Haran, Jarden and Sacerdote ([7,5,6]), played in Model Theory of finite and pseudofinite fields, providing a more precise form, as well as the effectivity of quantifier elimination. In this light, our work will have an impact in the study of exceptional difference polynomials, difference version of Davenport's problem, graphs definable in fields with Frobenii and existentially closed difference fields, and many other areas inspired by applications of the classical Galois stratification over finite and pseudofinite fields.

In papers [18] and [17], we developed a theory of twisted Galois stratification for generalised difference schemes, and we established a rather *fine quantifier elimination* result, stating that every first-order formula in the language of difference rings is equivalent to a *Galois formula* modulo the theory ACFA of existentially

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closed difference fields, where the latter formulae are associated with *finite* Galois covers of difference schemes. We argued that the elimination procedure was *effective* in the sense that it was primitive recursive reducible to a few natural operations in difference algebra (the status of which is unknown at the moment).

In this paper, we develop *direct twisted Galois stratification* in the context of direct presentations of difference schemes, which approximates the difference scheme framework to a sufficient order. We show a slightly coarser quantifier elimination result, Theorem 5.9, which (informally) states that

> every first-order formula is equivalent to a *direct Galois formula* modulo ACFA, or over the fields with Frobenii,

where the latter formulae are associated with direct Galois covers. Even though the class of direct Galois formulae is coarser than that of Galois formulae, direct Galois formulae are equivalent (4.16) to the $\exists_1$-formulae that appear after the known *logic quantifier elimination* for ACFA from [12] and [3].

Our main result (Theorems 6.7 and 6.10) is that

> the quantifier elimination procedure for ACFA and for fields with Frobenii is *primitive recursive*.

Given that working with direct presentations essentially reduces to working with algebraic varieties and correspondences between them, this follows by applying methods of classical effective/constructive algebraic geometry in our framework. Consequently, ACFA and the first-order theory of fields with Frobenii are decidable by a primitive recursive procedure, see Corollaries 6.8 and 6.12.

The present paper is not a variant of [18] and [17] since the whole machinery of direct Galois covers had to be developed from first principles, which is significantly more intricate than previous considerations involving difference schemes. There is no direct interaction with the methods of the previous papers, they only provide ideological guidance to identify the main conceptual steppingstones in the stratification procedure.

### *1.2. Direct presentations of difference schemes*

Let $(k, \varsigma)$ be a difference field, let $X$ be an algebraic variety over $k$, let $X_\varsigma$ denote the base change of $X$ via $\varsigma : k \to k$, and let $W \subseteq X \times X_\varsigma$ be a closed subvariety. Let $(F, \varphi)$ be any difference field extending $(k, \varsigma)$. The intuitive idea that sets of the form

$$\{x \in X(F) : (x, x\varphi) \in W\}$$

should correspond to sets of $(F, \varphi)$-points of a 'difference variety' has been around from the beginning of research on difference algebra, and it was particularly useful in the model-theoretic study of difference fields, as in [12] and [3].

Although the above data determines a 'directly presented difference scheme' defined in [10], we choose to minimise the use of the framework of difference schemes, and remain in the context of their *direct presentations*, using the classical language of algebraic schemes and correspondences (we only use difference schemes to control parameters, since the alternative leads to rather cumbersome notation). The benefit of this approach is that we can profit from the methods of effective algebraic geometry in order to prove that our constructions are primitive recursive.

### *1.3. Generalised difference schemes*

The logic quantifier elimination mentioned above states that a first-order formula in the language of difference rings is equivalent to an existential formula modulo ACFA. Intuitively, such a formula chooses

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