Assertion and denial: A contribution from logical notations

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A R T I C L E   I N F O

A B S T R A C T

This paper presents two major aspects of Frege's and Peirce's views on assertion and denial: first, their arguments for the notational choices concerning the representation of assertion and denial in Begriffsschrift (BS) and Existential Graphs (EGs), respectively; and second, those properties of BS and EGs which reflect their inventors' views on assertion and denial. We show that while Frege's notation has an *ad hoc* sign of assertion and an *ad hoc* sign of negation, Peirce has a sign of assertion which is also a sign of logical conjunction, and a sign of scope which is also a sign of negation.

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0. Introduction

What is an assertion, how does it differ from a proposition? What is a denial, and how does it differ from an assertion? Philosophers of language and logic have been occupied with such questions at least since Frege's distinction between assertion and the content asserted, a distinction which is expressed in his notation by means of a sign of assertion (the *Urteilsstrich*). Frege has also been credited with being among the firsts to have made the point that negation is not the polar opposite of assertion.

Yet Frege was not the first and not even the most consequential philosopher of language and logic pronouncing upon assertions and denials. Peirce had a number of points to be made on assertion and denial that call upon new and much belated investigation and assessment. Just like Frege, he invented a novel logical notation that expresses quantificational logic, yet one that in significant ways was different both from Frege's notation and from the notation that has become the received language of first-order logic.

Frege invented the Begriffsschrift (BS) in 1879 [11], Peirce the Existential Graphs (EGs) in 1896 [26–35]. The present paper investigates two major aspects of Frege's and Peirce's views on assertion and denial: first, their arguments for the notational choices in BS and EGs concerning the representation of assertion and denial; and second, those properties of BS and EGs which reflect their inventors' views on assertion

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and denial. Frege's and Peirce's notations differ from other logical notations as well as from each other, but the sense of this "difference" has to be closely examined. As far as the representation of assertion and denial is concerned, we show that while Frege's notation employs an *ad hoc* sign of assertion and an *ad hoc* sign of negation, Peirce has a sign of assertion which is *also* a sign of logical conjunction, and a sign of scope which is *also* a sign of negation. Though we limit our investigation to Frege's and Peirce's notational views on assertions and denial, and to their theories of the BS and EGs, respectively, we believe that our lesson is also a contribution to the contemporary and the more general and largely unresolved question of how equivalently expressive notations of a system of logic can differ in ways that are both philosophically significant and relevant to the development of logical and linguistic theories of meaning. Also, the limitation of our discussion to the analysis of assertions in the contexts of the two theories of the BS and EGs by no means implies that there is no interesting work that both Frege and Peirce did on assertions outside of these logical theories. For example, [1, 5, 6, 18, 20] have all discussed Peirce's responsibility-taking view on assertions without specific attention to the consequences of his theory of EGs in which propositions are expressed on the sheet of assertion.

The paper is divided in two sections, which are devoted to the topics of assertion and denial, respectively. Each section is in its turn divided into two subsections, devoted to Frege (subsections 1.1 and 2.1) and Peirce (subsections 1.2 and 2.2) on assertion and denial.

1. Assertion

1.1. Frege on assertion

Peter Geach [16, p. 449] has devoted considerable attention to the thesis that 'a thought may have just the same content whether you assent to its truth or not; a proposition may occur in discourse now asserted, now unasserted, and yet be recognizably the same proposition'. Geach terms this the 'Frege Point'. Another formulation of the Frege Point is by David Bell: 'propositional identity survives changes in assertiveness' [3, p. 92]. The Frege Point, Geach argued, is something we need in order to understand *modus ponens*:

(1) (i) 'If p, then q'
     (ii) 'But p'
     (iii) 'Therefore, q'

In order to understand an argument of the form (1), we need to assume that it is one and the same proposition, 'p', that occurs asserted in (1.ii) but not in (1.i). The reason is that if 'p' occurred asserted in the conditional (1.i), then (1.ii), which simply asserts 'p', would contain nothing not contained in (1.i), and thus would be redundant. On the other hand, if the proposition 'p' that occurs asserted in (1.ii) were not in some sense the same as the proposition 'p' that occurs in (1.i), the argument would be vitiated by equivocation. Thus, in order to reconcile these two facts, we need to assume that while 'p' is the same proposition in both (1.i) and (1.ii), yet it occurs asserted in (1.ii) but unasserted in (1.i). And thus the point that a proposition or propositional content may occur in logical discourse now asserted, now unasserted, is established. For reasons that will become apparent in the present paper, we propose to re-label Geach's Frege point as the 'Geach Point'.

Besides maintaining the Geach Point, namely that one and the same proposition may occur asserted in some contexts and unasserted in others, Frege also maintained that 'the distinction between asserted and unasserted occurrence [has to] be exhibited notationally' [8, p. 152]. We call the thesis that the difference between the case in which 'p' occurs asserted and the case in which 'p' occurs unasserted in some context has to be notationally expressed, *whatever form the notational expression of this difference happens to take*, the 'Dudman Point'. Frege thought the Geach Point to entail the Dudman Point, and for that reason endowed