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On assertion and denial in the logic for pragmatics

Massimiliano Carrara [a], Daniele Chiffi [b], Ciro De Florio [c]

[a] Padua University, Italy
[b] Tallinn University of Technology, Estonia
[c] Catholic University of Milan, Italy

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ABSTRACT

The aim of this paper is twofold: First, we present and develop a system of logic for pragmatics including the act of *denial*. Second, we analyse in our framework the so-called paradox of assertability. We show that it is possible to yield sentences that are not assertable. Moreover, under certain conditions, a symmetric result can be obtained: There is a specular paradox of deniability. However, this paradox is based on the problematic principle of classical denial equivalence.

1. Introduction

In the *classical theory for denial*, to deny A is equivalent to asserting $\neg A$:

Classical denial. A is correctly denied *iff* $\neg A$ is correctly asserted.

In Ripley [21] the above equivalence is called *denial equivalence.*[1] *Classical denial* implies what we call the *Frege's Thesis, i.e.* that there is just one speech act, *i.e.* assertion, and that denial is reducible to it. The aim of the paper is to defend the thesis that, from the assertion of the negation of A, it is possible to infer the denial of A, but not *vice versa*. Frege's Thesis does not hold. We will argue that, if the act of asserting is embedded in a general framework of pragmatic logic, then it is plausible to consider assertion and denial as relatively independent speech acts governed by different conditions of justification. We show that this view provides a suitable framework for dealing with some pragmatic paradoxes similar to the liar, the *assertability paradox* and the *deniability paradox*. We first introduce Dalla Pozza and Garola's *pragmatic logic* (LP) (section 2) where a *logic of assertion* is *formulated*. Then, we argue (section 3) against Frege's Thesis. We propose a basic, intuitive extension of LP with denial, considering *assertion* and *denial*

E-mail address: massimiliano.carrara@unipd.it (M. Carrara).

[1] For general background on denial in non-classical theories, see (Ripley [21], §3). On the same topic see also Restall [20].

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(or rejection) as two relatively independent speech acts. Within such an extension, the *assertability paradox* and the *deniability paradox* (section 5) are analyzed.

2. A pragmatic logic for assertion (LP)

In the logical system called *Logic for Pragmatics* (LP), Dalla Pozza and Garola [9] provided a formal treatment of assertion by introducing some pragmatic connectives and the sign of *force*, which are required to formulate a pragmatic interpretation of intuitionistic propositional logic as a logic of assertions.[2]

Assertions are intended as "purely logical entities ... without making reference to the speaker's intention or beliefs" (Dalla Pozza and Garola [9], 83). LP is composed of two sets of formulas: *radical* and *sentential*. Every sentential formula contains at least a radical formula as a proper sub-formula.

Radical formulas are semantically interpreted by assigning them (classical) truth-values. *Sentential formulas* (briefly, *assertions*), on the other hand, are pragmatically evaluated by assigning them justification values defined in terms of the intuitive notion of proof. In other words, LP propositions can be either true or false, whereas the judgements expressed as assertions can be justified (J) or unjustified (U). The pragmatic language of LP is described below.

Alphabet.

The vocabulary of LP contains the following sets of signs.

Descriptive signs: the propositional letters p, q, r, ...
Logical signs for radical formulas: $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$
Logical signs for sentential formulas: the assertion sign $\vdash$ and the pragmatic connectives $\sim$ (negation), $\cap$ (conjunction), $\cup$ (disjunction), $\supset$ (implication), $\equiv$ (equivalence).

Formation rules (FRs)
Radical formulas (*rfs*) are recursively defined by the following FRs.
FR1 (atomic formulas): Every propositional letter is an *rf*.
FR2 (molecular formulas):
(i) Let γ be an *rf*; then $\neg\gamma$ is a *rf*;
(ii) Let γ_1 and γ_2 be *rfs*; then $\gamma_1 \wedge \gamma_2$, $\gamma_1 \vee \gamma_2, \gamma_1 \rightarrow \gamma_2$, $\gamma_1 \leftrightarrow \gamma_2$ are *rfs*.

Sentential formulas (*sfs*) are recursively defined by the following FRs.
FR3 (elementary formulas): Let γ be an *rf*, then $\vdash \gamma$ is an *sf*.
FR4 (complex formulas):
(i) Let δ be an *sf*; then $\sim \delta$ is an *sf*;
(ii) Let δ_1 and δ_2 be *sfs*; then $\delta_1 \cap \delta_2$, $\delta_1 \cup \delta_2$, $\delta_1 \supset \delta_2$, $\delta_1 \equiv \delta_2$ are *sfs*.

Every radical formula of LP has a truth-value. Every sentential formula has a justification value, which is defined in terms of the intuitive notion of proof and depends on the truth-value of its radical sub-formulas. The semantics of LP is the same as for classical logic, and it provides only the interpretation of the radical formulas by assigning them truth-values and taking propositional connectives as truth functions in a standard way.

To be precise, the semantic rules are the usual classical Tarskian ones and specify the truth-conditions (only for radical formulas) through assignment function σ, thus regulating the semantic interpretation of LP. Let γ_1, γ_2 be radical formulas and $1 =$ true and $0 =$ false; then:

[2] For an extension of LP see Carrara et al. [6,7], Bellin et al. [2,3].

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