ARTICLE IN PRESS
JAL:478

Journal of Applied Logic ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Applied Logic

www.elsevier.com/locate/jal

Bilateralism does not provide a proof theoretic treatment of classical logic (for technical reasons)

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ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Bilateralism
Proof theoretic justification
Harmony
Normalisation

ABSTRACT

In this short paper I note that a key metatheorem does not hold for the bilateralist inferential framework: harmony does not entail consistency. I conclude that the requirement of harmony will not suffice for a bilateralist to maintain a proof theoretic account of classical logic. I conclude that a proof theoretic account of meaning based on the bilateralist framework has no natural way of distinguishing legitimate definitional inference rules from illegitimate ones (such as those for *tonk*). Finally, as an appendix to the main argument, I propose an alternative non-bilateral formal solution to the problem of providing a proof-theoretic account of classical logic.

1. Introduction

I assume that the basics of the various proof theoretic theories of logical constants, or justifications of deduction, are familiar to the reader.

To recap briefly, a proof theoretic account typically takes it as given that we use certain basic inference rules for the logical constants and account for their validity and our awareness (and knowledge) of them in terms of a suitable proof theoretic theory of meaning; it then shows that the logical truths can be derived from these inference rules; it has then accounted for deduction, logical truth and the relation between the two.

A familiar way of presenting this idea is to argue, frequently in the context of Gentzen–Prawitz Natural Deduction (henceforth simply ‘Natural Deduction’),[2] that the meanings of the logical connectives are given proof theoretically, defined by their introduction and/or elimination rules. This idea can be traced back to a remark by Gentzen in [5, p. 80]

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[1] I am grateful to the anonymous referees for their helpful remarks.
[2] For the ‘Prawitz’ part see [9], for the ‘Gentzen’ part see [16].

https://doi.org/10.1016/j.jal.2017.11.001

> ... The introductions represent, as it were, the 'definitions' of the symbol concerned, and the eliminations are no more, in the final analysis, than the consequences of these definitions. ...

This idea has been taken on in many different forms by a number of different authors, but all variations must respond to Prior's famous observation in [10] that not all inference rules will do as definitions of logical constants. Prior's connective *tonk* is 'defined' so that

$$\frac{A}{A\ tonk\ B}\ tonk\, I \qquad \frac{A\ tonk\ B}{B}\ tonk\, E$$

are valid. But in a system with these inference rules all propositions become inter-derivable. For the proof theoretic account to work, some explanation must be given of why, e.g., the familiar inference rules for conjunction are good, but the inference rules for *tonk* are bad.

Remaining within the context of Natural Deduction, a standard suggested requirement on a system of proof theoretic definitions of logical connectives is that its derivations normalise. That is, in any derivation, no connective is required to be introduced and then later be eliminated. This is often termed as a requirement that the introduction and elimination rules be in *harmony*. I shall assume that the reader has a familiarity with these proof theoretic concepts.

The requirement of harmony is simple, intuitive and can be well motivated. It is not simply *ad hoc* as a means of solving the problem of *tonk*. For example, for Dummett in [2], the requirement of harmony derives from a verificationist theory of meaning, a theory which has its own independent set of arguments (such as the so called 'manifestation argument' outlined by Dummett in briefer form in [1]). I shall not enter a discussion of the various motivations for the requirement of harmony.[3] It is enough here to point out that it not only rules out *tonk* as a legitimately defined connective, but also classical proof theory (again in the context of Natural Deduction). What is left is intuitionistic proof theory. We then arrive at the issue at hand: can a proof theoretic account of the logical constants be modified to account for classical logic, or is it forced into intuitionism?

2. Bilateralism

Rumfitt in [12], following a suggestion of Smiley in [13], proposes that the structure of Natural Deduction fails to take into account that falsity is as important as truth, or that denial as important as assertion. In other words, we should see the inference rules that define a connective as characterising not just its derivation conditions, but also its refutation conditions. From the proof theoretic perspective, this proposal amounts to the observation that a proposition in a derivation should have one of two polarities (for 'derived' and 'rejected', or more simply for 'yes' and 'no').

Rumfitt enhances Natural Deduction with the addition of polarities to the nodes in the derivation tree. The polarities are signified by with $+$ or $-$. With these modifications the derivation rules for, e.g. negation become:

$$\frac{+A}{-\neg A}\ -\neg I \quad \frac{-A}{+\neg A}\ +\neg I \qquad \frac{-\neg A}{+A}\ -\neg E \quad \frac{+\neg A}{-A}\ +\neg E$$

[3] A substantial discussion is contained in the recent book [4], which also discusses the variations on Natural Deduction that form the context for the notion of harmony.

Please cite this article in press as: M. Gabbay, Bilateralism does not provide a proof theoretic treatment of classical logic (for technical reasons), J. Appl. Log. (2017), https://doi.org/10.1016/j.jal.2017.11.001